Measuring Oscillating Axion Fields Using Paul Traps

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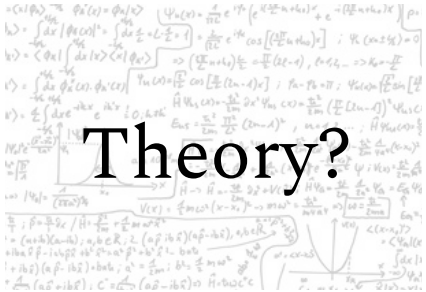
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Institute for Theoretical Physics

2023

Introduction

Strong CP problem?



Theory?

AXIONS?

Dark Matter?

Sensing?

Introduction

Classical type detectors aren't sensitive enough!

$m_a < 10^{-6}$ eV

$m_{\text{neutrino}} < 0.12$ eV

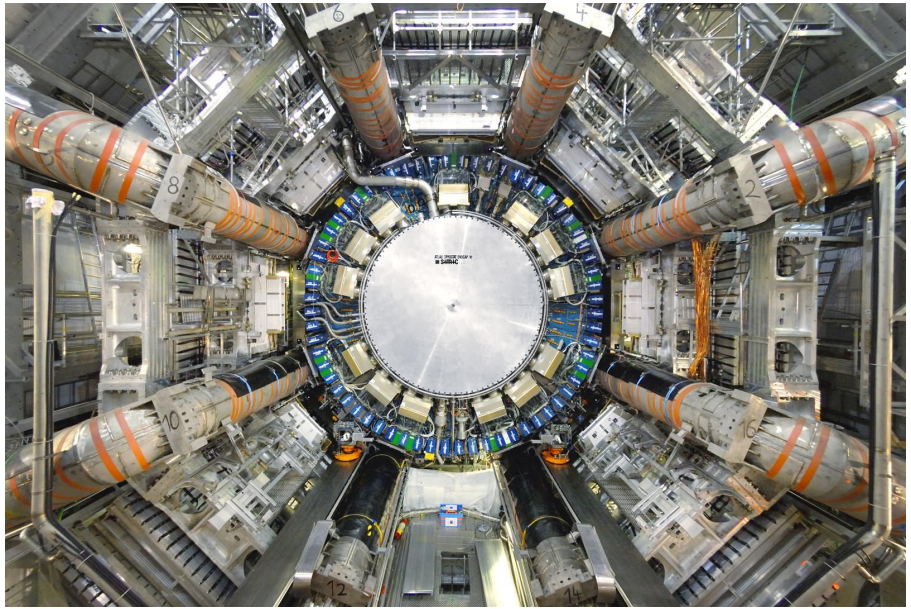


Image: ATLAS, CERN

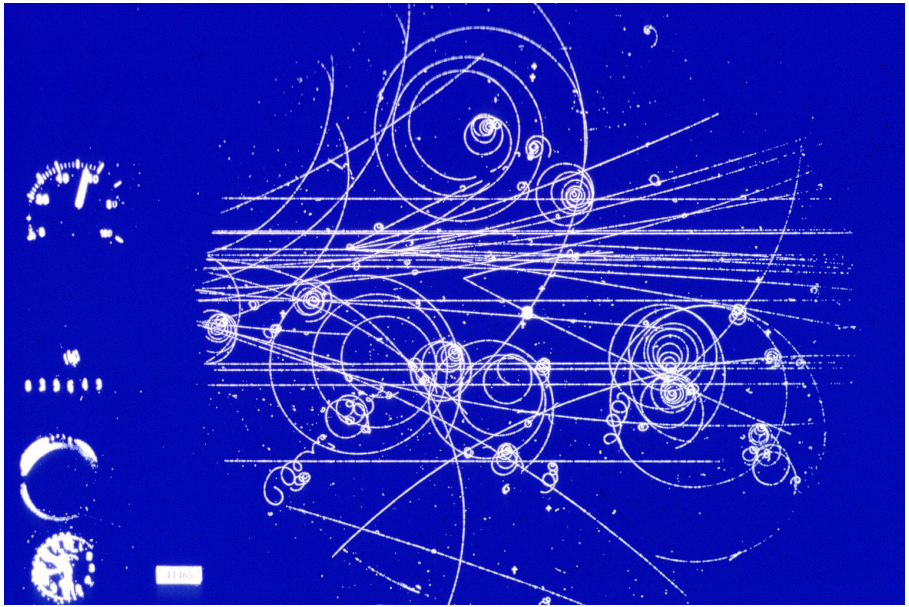


Image: "The decay of a lambda particle in the 32 cm hydrogen bubble chamber" CERN

Introduction

Paul traps

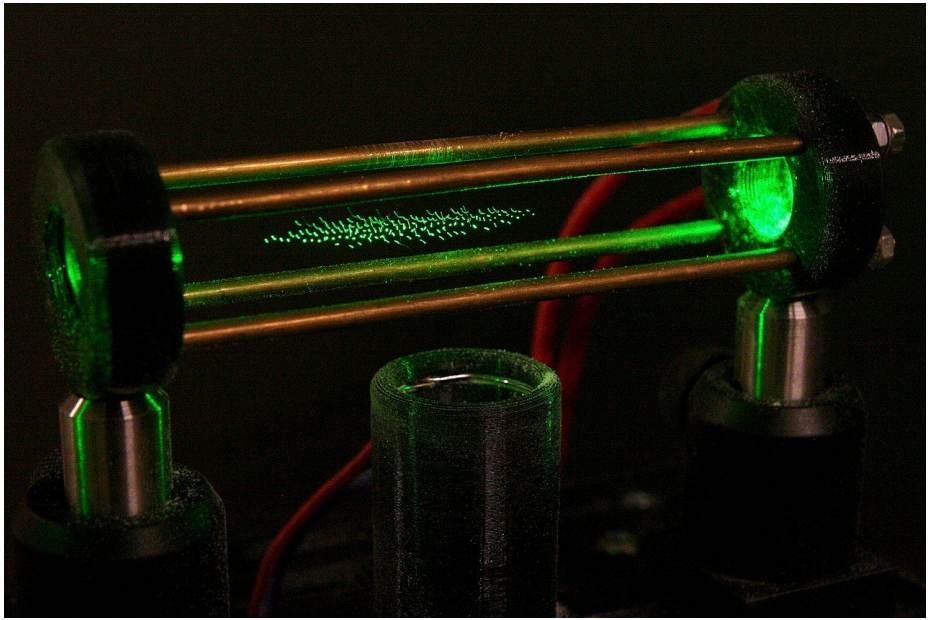


Image: Jan Pavelka on Wikipedia

Introduction

Axions exist

⌄

Their field interacts with ions

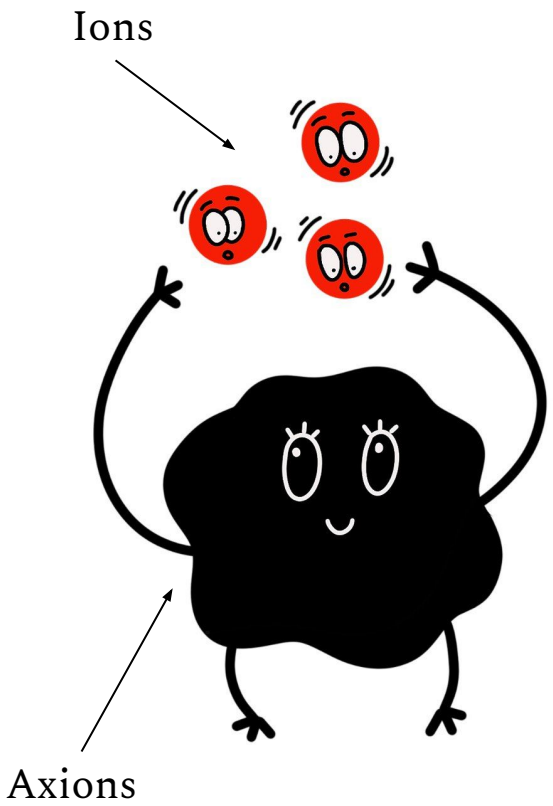
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We measure this interaction

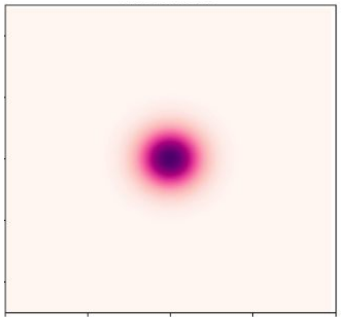
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We know parameters of axions

(hopefully)

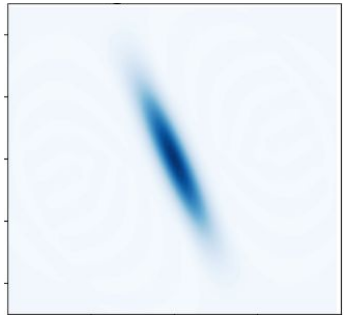


Introduction



"Classical" approach

Increased sensitivity by adding more ions or getting rid of the noise



"Quantum" approach

Increased sensitivity by using entanglement/spin squeezed states

Axion electrodynamics

Strong CP problem

$$L_{QCD} = -\frac{1}{4}\underbrace{G^a_{\mu\nu}G^{\mu\nu}_a}_{CP\checkmark} + \frac{\alpha_s}{8\pi}\bar{\theta}\underbrace{G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a}_{CP\times} + \underbrace{\bar{\psi}_i(i\gamma^\mu D_\mu - M)\psi_i}_{CP\checkmark\times 2}$$

- Non-zero dipole moment for the neutron: $d_n = (2.4 \pm 1.0)\theta \times 10^{-3}$ e fm
- Experimentally: $|d_n| < 1.8 \times 10^{-13}$ e fm
- Stringent upper limit for $|\bar{\theta}|$: 0.8×10^{-10}

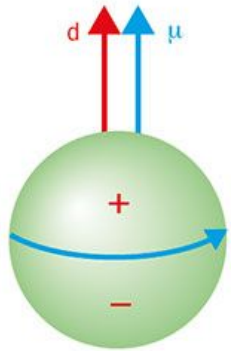


Image: "Storage ring steps up search for electric dipole moments" CERN Courier

Axion electrodynamics

The Peccei-Quinn mechanism 1977

$$L_{QCD+a} = L_{QCD} + \frac{\alpha_s}{8\pi}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

- Addition of the new *U(1)* symmetry that breaks at high energy scale
- At long times axion field absorbs theta term

But! New particle axion, which has not been yet discovered experimentally

$$m_a \approx 6\mu eV\left(\frac{10^{12}GeV}{f_a}\right)$$

Dark Matter candidate?



Image: amazon.com

Axion electrodynamics

Coupling to EM fields

- Oscillating axion field

$$a(t) = a_0 \cos(m_a t)$$
$$\rho_{DM} = \rho_a \approx 1/2 m_a^2 a_0^2$$

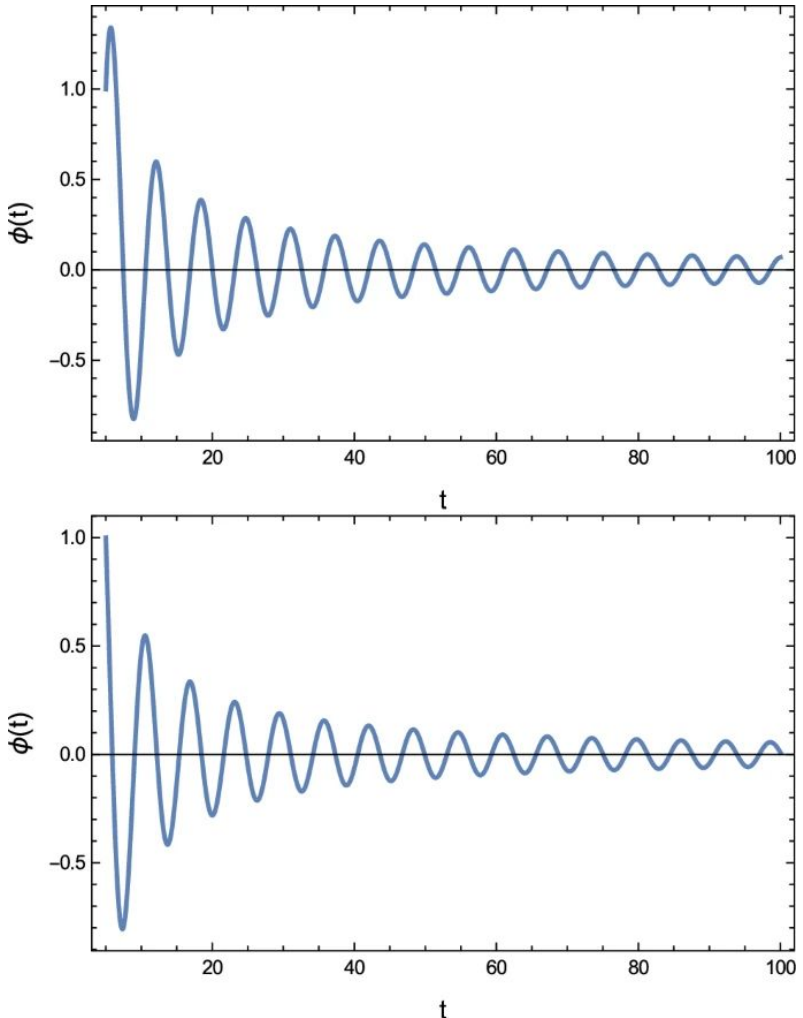


Image: "Collapse of an axion scalar field" Soumya Chakrabarti

Axion electrodynamics

Coupling to EM fields

CP-even form

$$L_{a\gamma}^{\text{CP-even}} = -\frac{g_{a\gamma}}{4} a \tilde{F}_{\mu\nu} F^{\mu\nu} = g_{a\gamma} a \frac{1}{c} \mathbf{E} \cdot \mathbf{B}$$

CP-odd form

$$L_{a\gamma}^{\text{CP-odd}} = \frac{\bar{g}_{a\gamma}}{4} a F_{\mu\nu} F^{\mu\nu} = \frac{2}{c^2} \bar{g}_{a\gamma} a (\mathbf{E}^2 - \mathbf{B}^2)$$

Consequence: time-oscillating values for EM constants

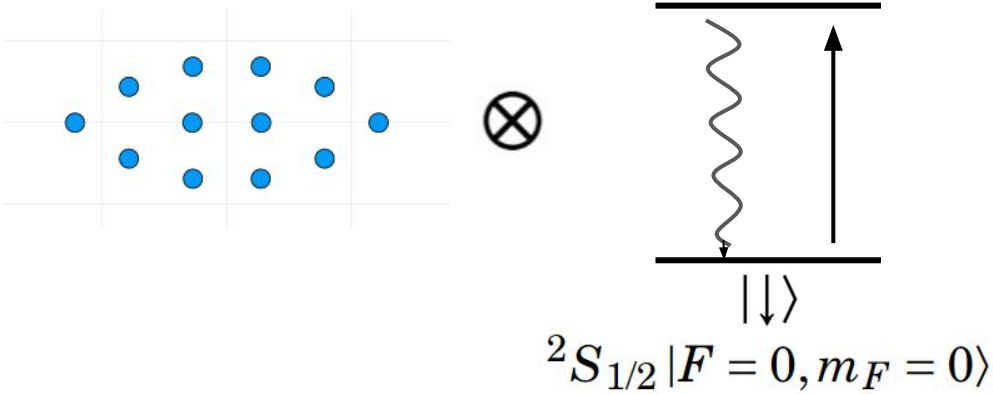
$$\alpha(t) \approx \alpha_{em}(1 + \bar{g}_{a\gamma} a(t))$$

axion-photon coupling constant

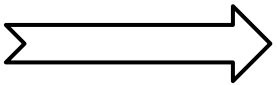
$$e \approx \underbrace{\sqrt{4\pi\epsilon_0 \hbar c \cdot \alpha_{em}}}_{e_{em}} \sqrt{1 + \bar{g}_{a\gamma} a_0 \cos(m_a t)}$$

Sensing with ions

Couple motion and spin states!



Axion field effects the motion of ions



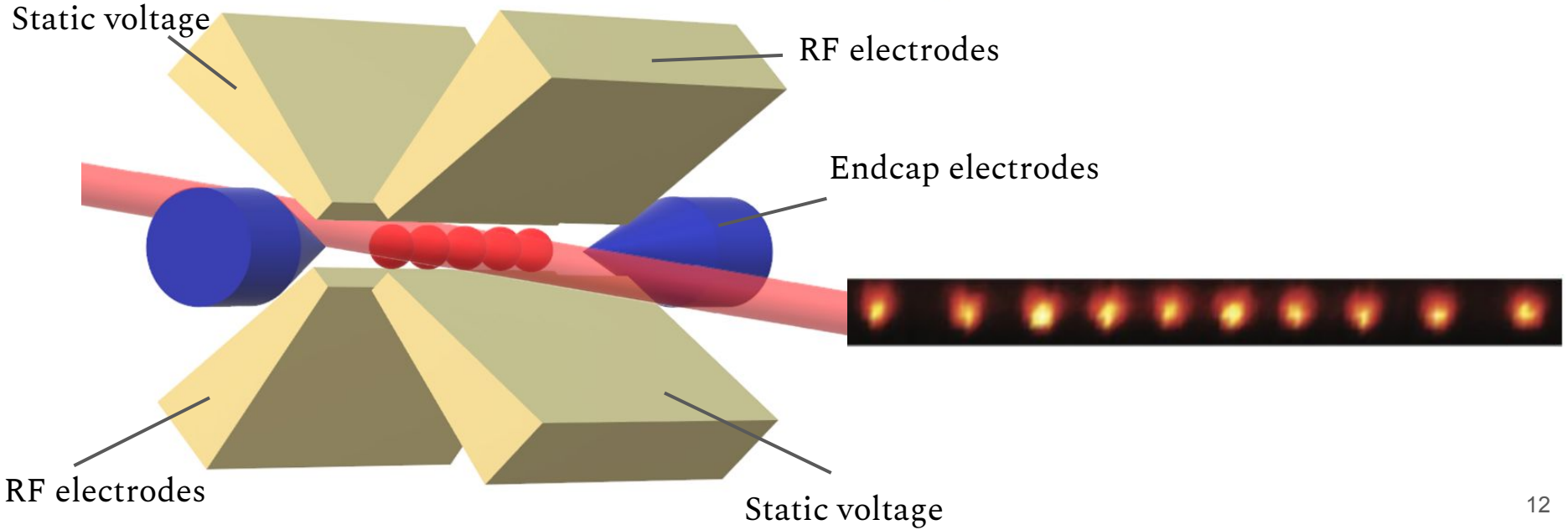
The number of ions in the "up" state is changed!

Ion crystals

Trap potential

Paul traps:

- Electrostatic + oscillating electric fields $V = \sum_{i=1}^{N} \frac{1}{2} M(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2)$



Ion crystals

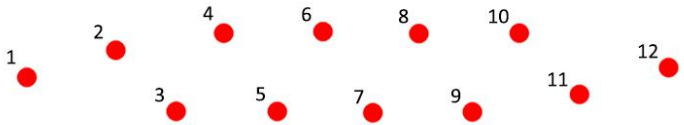
Trap potential

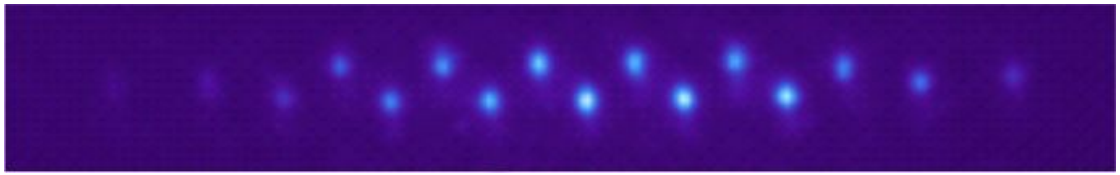
Paul traps:

$$V = \sum_{i=1}^{N} \frac{1}{2} M(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2) + \sum_{i,j=1, j\neq i}^{N} \frac{Z^2 e^2}{8\pi\epsilon_0} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Electrostatic + oscillating electric fields

Coulomb repulsion



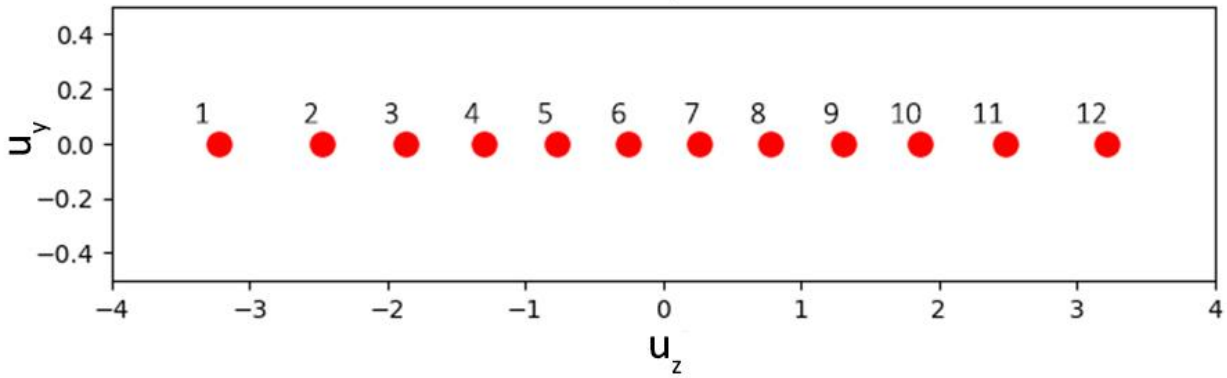


Ion crystals

Equilibrium positions

Assume ions are sufficiently cold: $\mathbf{r}_i(t) \approx \mathbf{r}_i^{(0)} + \mathbf{q}_i(t)$

Solve: $\nabla V = 0|_{\mathbf{r}_i = \mathbf{r}_i^{(0)}}$

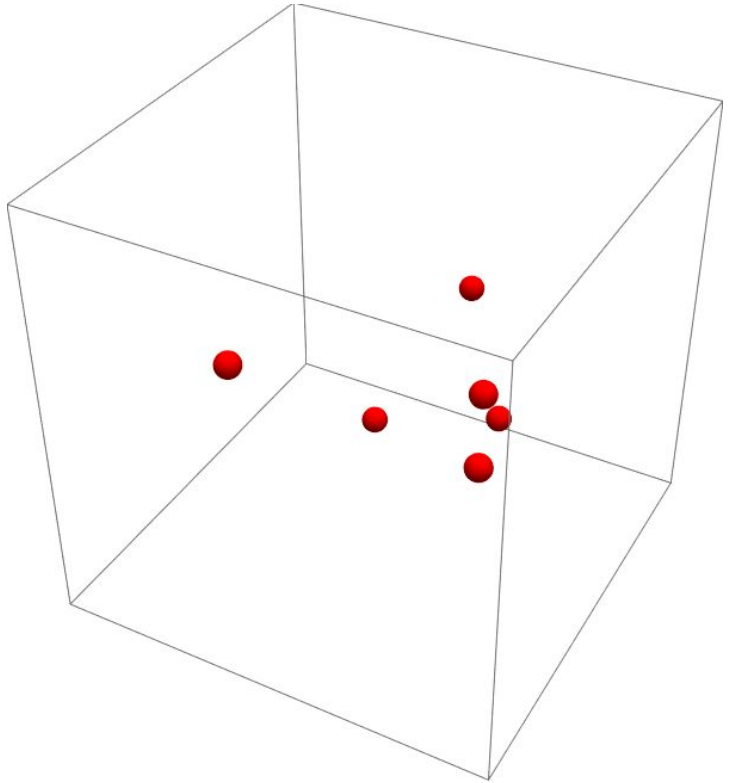


Ion crystals

Equilibrium positions

To approximate the eq. positions solve the equations of motion:

$$M\frac{d^2\mathbf{r}_i}{dt^2} = \mathbf{F}_{\text{trap}}(\mathbf{r}_i) + \sum_{j\neq i}\mathbf{F}_{\text{coul}}(i,j) + \mathbf{F}_{\text{cool}}$$



Ion crystals

Motional modes

Lagrangian of the system:

$$L = \frac{M}{2}\sum_{i=1}^{N}\sum_{\alpha=1}^{3}\dot{q}_{\alpha,i}(t) - \frac{1}{2}\sum_{i,j=1}^{N}\sum_{\alpha,\beta=1}^{3}\underbrace{\frac{\partial^2 V}{\partial u_{\alpha,i}\partial u_{\beta,j}}\Big|_{\substack{q_{\alpha,i}=0\\ q_{\beta,j}=0}}}_{\omega\cdot A_{i,j}^{\alpha,\beta}} q_{\alpha,i} q_{\beta,j}$$

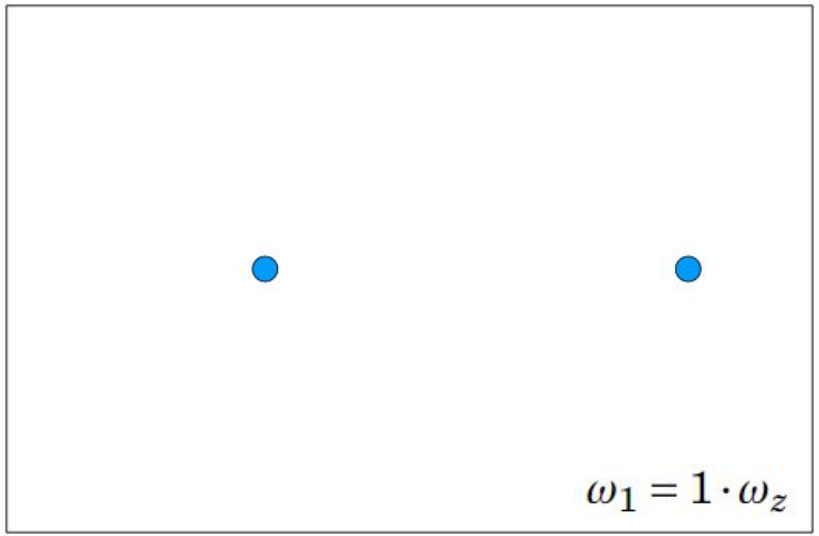
Eigenvectors $\mathbf{b}^{(m)}$ determine the normal modes of the ions corresponding to the mth mode

Eigenvalues μ_m are squares of frequencies of mth mode $\omega_m = \sqrt{\mu_m}\omega$

Ion crystals

Motional modes

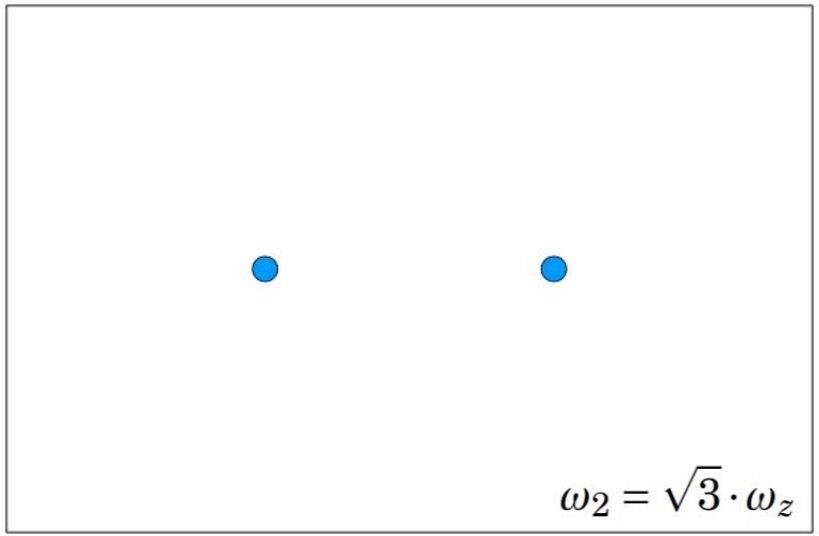
$m = 1$



Center-of-mass (COM) mode

$$\mathbf{b}_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$m = 2$



Breathing mode

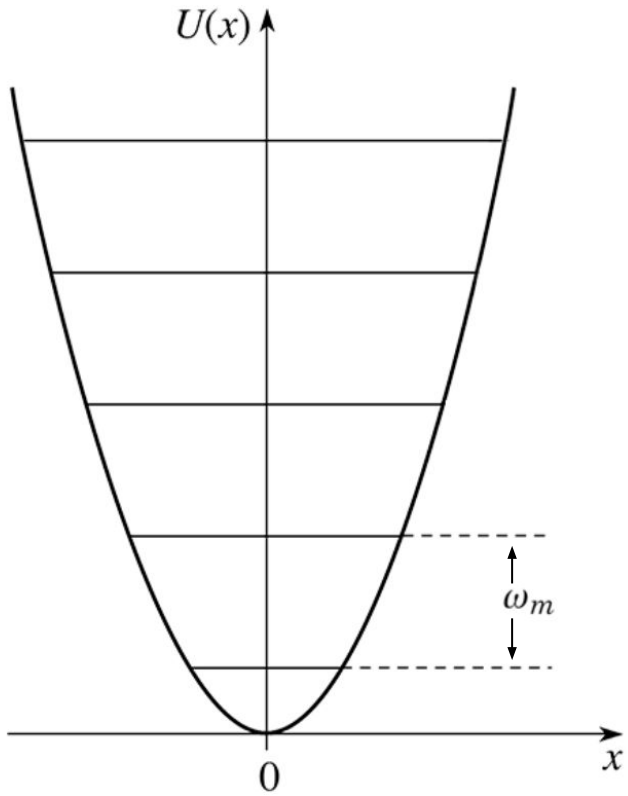
$$\mathbf{b}_2 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

Ion crystals

Motional modes

Hamiltonian in terms of motional modes:

$$\hat{H}_{\text{motion}} = \sum_{m=1}^{3N} \hbar\omega_m \left(\hat{a}_m^\dagger \hat{a}_m + \frac{1}{2} \right)$$



Spin dependent forces

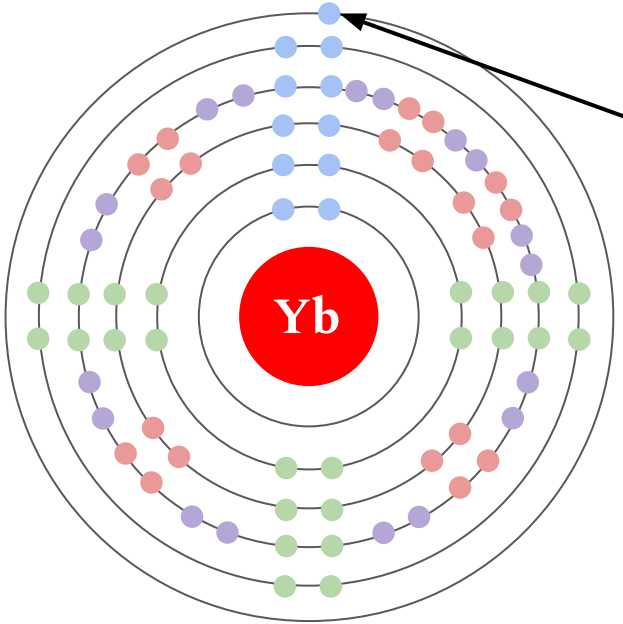
^{171}Yb+

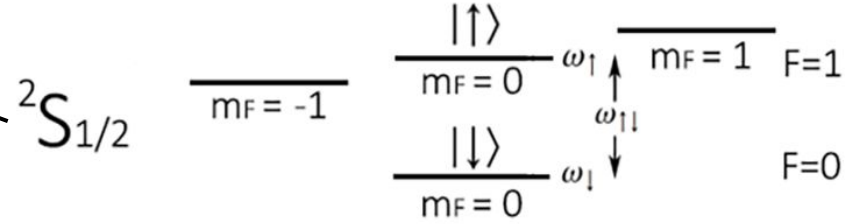
1. resilient to the effect of magnetic fields,
2. a narrow optical transition linewidth,
3. long coherence time

Define our "spin":

$^2S_{1/2}|F=1,m_F=0\rangle \longrightarrow |\uparrow\rangle$

$^2S_{1/2}|F=0,m_F=0\rangle \longrightarrow |\downarrow\rangle$





Spin dependent forces

Laser-ion interaction

Effective Hamiltonian:

$$\hat{H}_{\text{eff}}^{\text{all}} = -\sum_{i=1}^{N} \frac{\hbar\Omega_i}{2}\left(e^{-i(\mathbf{\Delta k_i}\cdot\mathbf{r_i}+[\omega_{\uparrow\downarrow}-\delta\omega]t-\phi_i)}\hat{\sigma}_-^i + e^{i(\mathbf{\Delta k_i}\cdot\mathbf{r_i}+[\omega_{\uparrow\downarrow}-\delta\omega]t-\phi_i)}\hat{\sigma}_+^i\right)$$

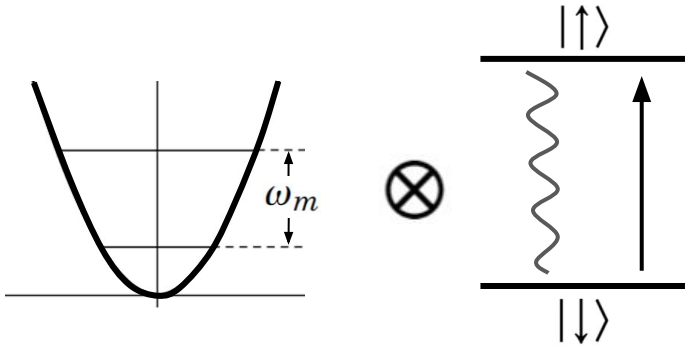
Ω_i – Rabi frequency strength

$\mathbf{\Delta k_i}$ – difference in momenta of the laser beams

$\mathbf{r_i}$ – position of an ion

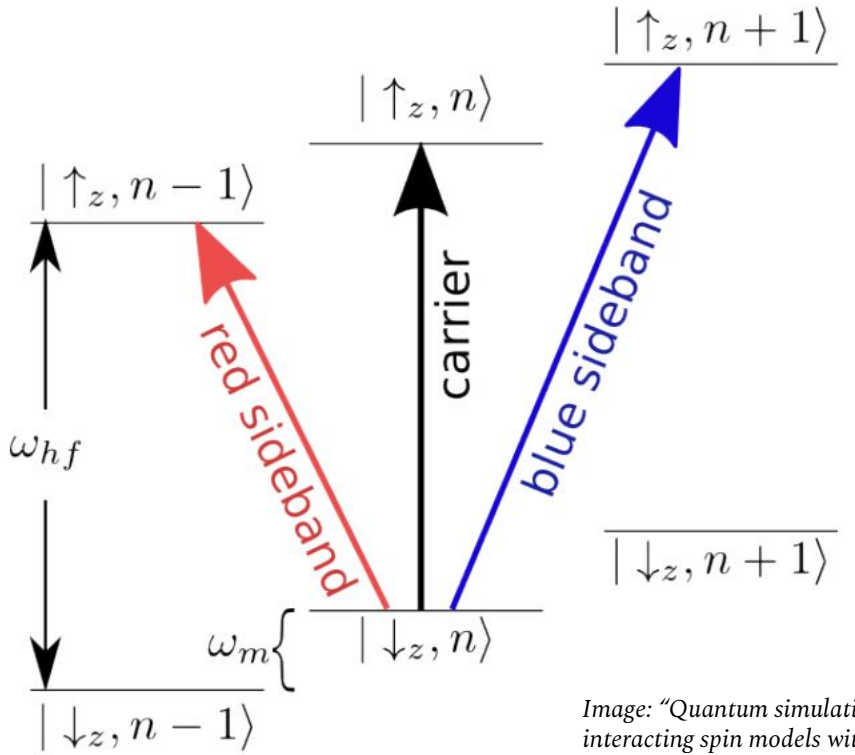
$\sigma_+ = |\uparrow\rangle\langle\downarrow|$

$\sigma_- = |\downarrow\rangle\langle\uparrow|$ – spin polarization states



Spin dependent forces

Manipulation of spin states - sideband transitions



Lamb-Dicke regime:
transition in motional state by $\Delta n = \pm 1$

$$\hat{H}_I^{i\prime} = \frac{\eta_{m,i}\hbar\Omega_i}{2}(\hat{a}_m e^{i\delta_m t} + \hat{a}_m^\dagger e^{-i\delta_m t})\hat{\sigma}_z^i$$

Image: "Quantum simulation of interacting spin models with trapped ions" K. Islam

Force sensing

"Classical" approach

Amplitude sensing below the zero-point fluctuations with a two-dimensional trapped-ion mechanical oscillator

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[2] *JILA and Department of Physics, University of Colorado, Boulder, Colorado, 80309, USA*
[3] *Georgia Tech Research Institute, Atlanta, Georgia 30332, USA*
[4] *U.S. Army Research Laboratory, Adelphi, Maryland 20783, USA*

(Dated: July 3, 2017)

We present a technique to measure the amplitude of a center-of-mass (COM) motion of a two-dimensional ion crystal of ∼100 ions. By sensing motion at frequencies far from the COM resonance frequency, we experimentally determine the technique's measurement imprecision. We resolve amplitudes as small as 50 pm, 40 times smaller than the COM mode zero-point fluctuations. The technique employs a spin-dependent, optical-dipole force to couple the mechanical oscillation to the electron spins of the trapped ions, enabling a measurement of one quadrature of the COM motion through a readout of the spin state. We demonstrate sensitivity limits set by spin projection noise and spin decoherence due to off-resonant light scattering. When performed on resonance with the COM mode frequency, the technique demonstrated here can enable the detection of extremely weak forces (< 1 yN) and electric fields (< 1 nV/m), providing an opportunity to probe quantum sensing limits and search for physics beyond the standard model.

Force sensing

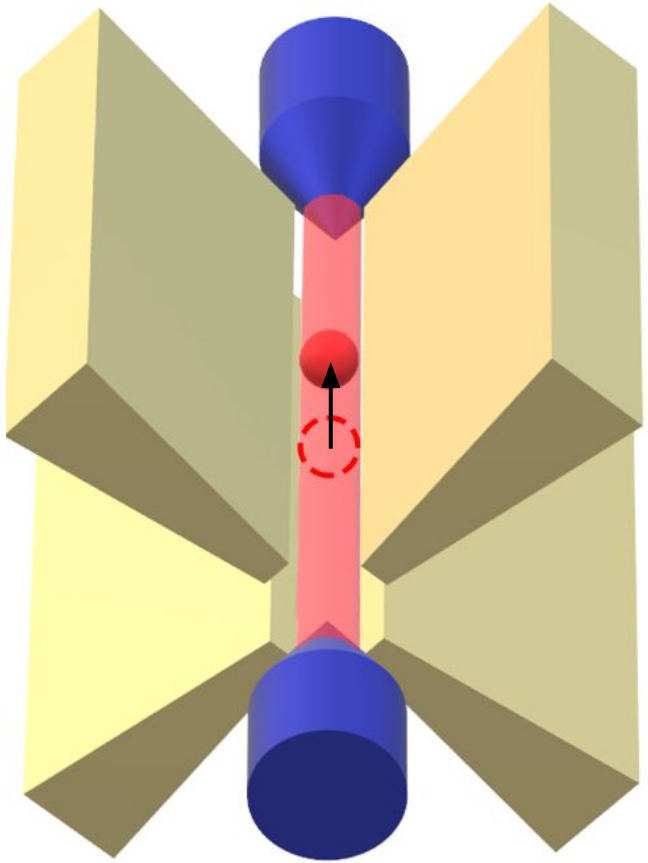
"Classical" approach

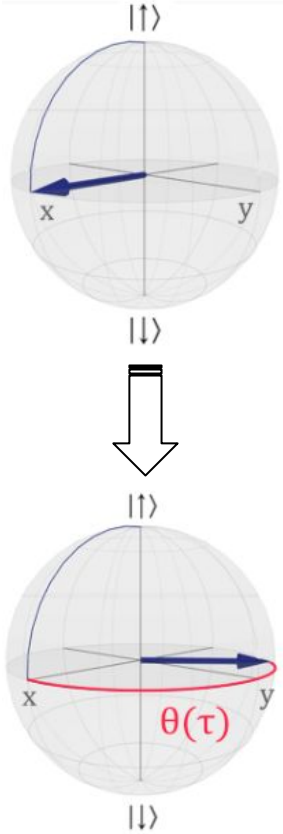
1. Couple spin and motion

2. Outside electric force, which has frequency far from COM resonance

- Displacement of ions

- Spin precession





Force sensing

Optical-dipole force

$$\hat{H}_{\mathrm{ODF}} = \sum_{i=1}^{N} F_0 \cos(\mu t)\hat{z}_i\hat{\sigma}_z^i \iff \hat{H}_I' = \sum_{i=1}^{N}\sum_{m=1}^{N} \frac{\eta_{m,i}\hbar\Omega_i}{2}(\hat{a}_m e^{i\delta_m t} + \hat{a}_m^\dagger e^{-i\delta_m t})\hat{\sigma}_z^i$$

$$\delta_m = \mu - \omega_m$$

$\hat{z}_i$ – ion position operator

$\hat{\sigma}_z^i$ – Pauli spin matrix for ith ion

μ – beatnote frequency

F_0 – ODF magnitude

! ODF pulse has frequency μ
far from COM resonance frequency ω_z

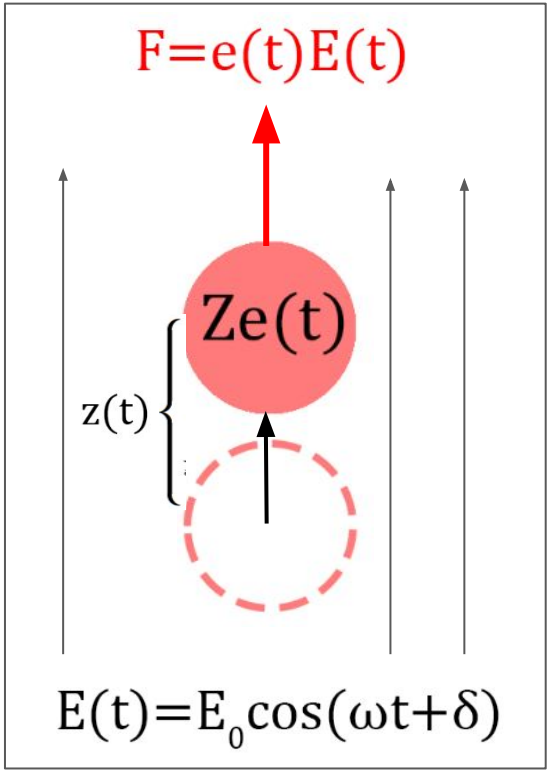
Force sensing

Optical-dipole force

$$\hat{H}_{\mathrm{ODF}} = \sum_{i=1}^{N} F_0 \cos(\mu t)\hat{z}_i \hat{\sigma}_z^i$$

Apply external electric field, so that $\hat{z}_i \rightarrow \hat{z}_i + z(t)$

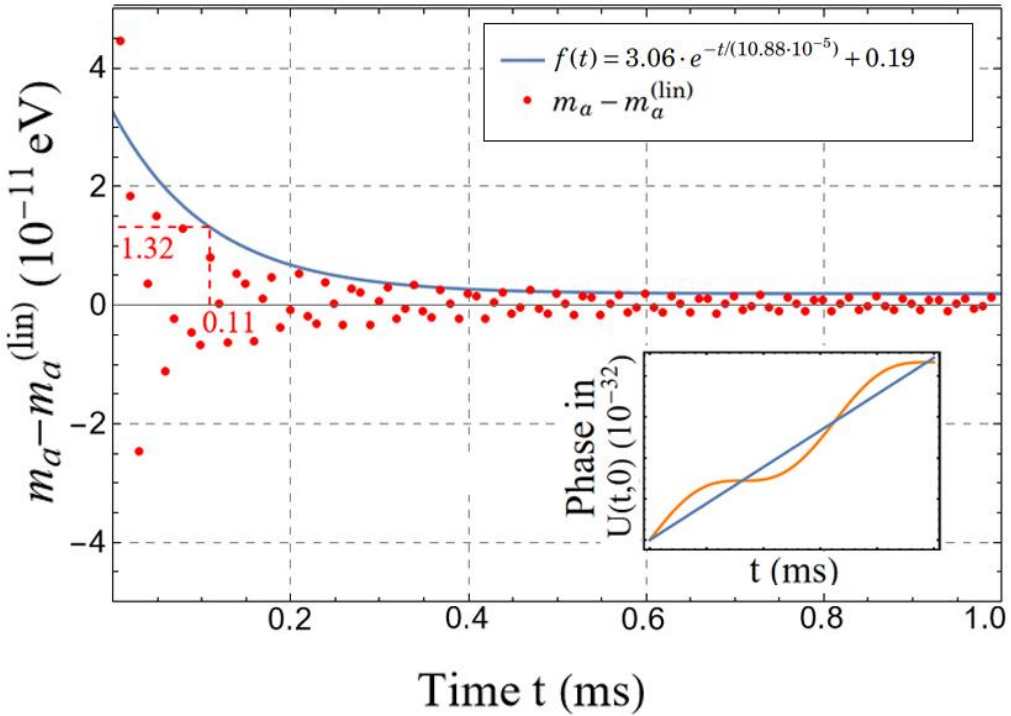
$$z(t) = \frac{Ze_{em}E_0}{M}\left[\frac{\cos(\omega t+\delta)}{\omega^2}\right.$$
$$\left. + \frac{\bar{g}_{\alpha\gamma}a_0}{4}\frac{\cos((\omega-m_a)t+\delta)}{(\omega-m_a)^2} + \frac{\bar{g}_{\alpha\gamma}a_0}{4}\frac{\cos((\omega+m_a)t+\delta)}{(\omega+m_a)^2}\right]$$



Force sensing

Optical-dipole force

$$\hat{H}_{ODF} \approx F_0 \frac{Z e_{em} E_0}{4M} \frac{\bar{g}_{\alpha\gamma} a_0}{(\omega - m_a)^2} \left(\cos\left((\omega - m_a - \mu)t + \delta\right)\right) \sum_{i=1}^{N} \frac{\hat{\sigma}_z^i}{2}$$



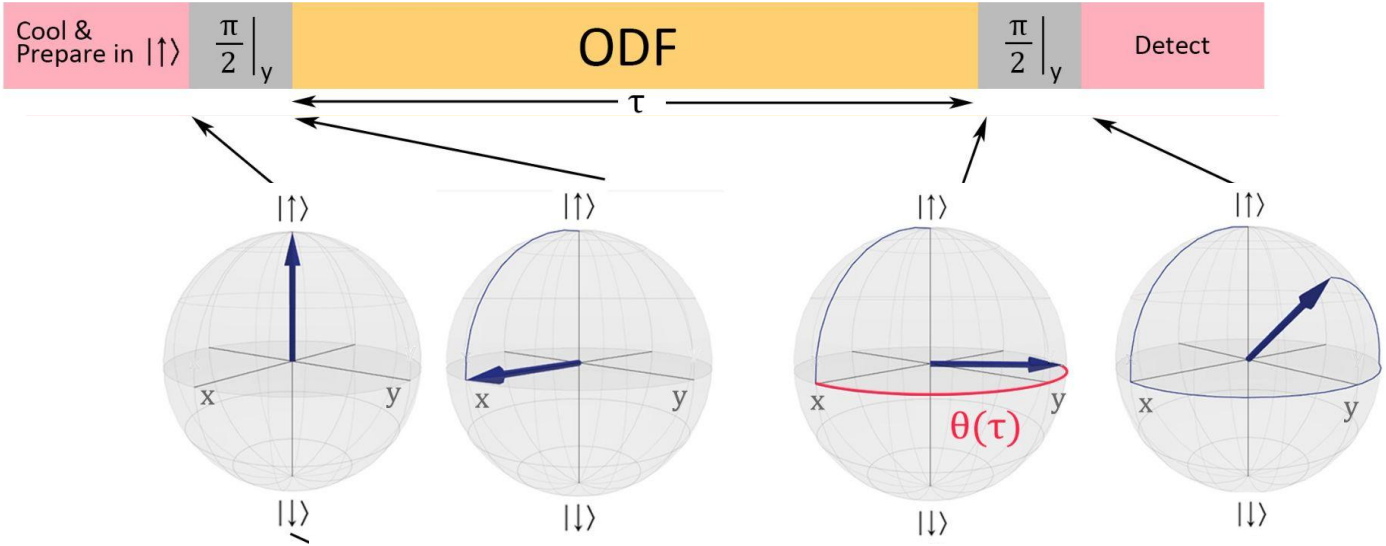
At τ=20 ms approximation error is 2%

Amplitude of displacement:

$$Z_c = \frac{Z e_{em} E_0}{4M} \frac{\bar{g}_{\alpha\gamma} a_0}{(\omega - m_a)^2}$$

Force sensing

Protocol

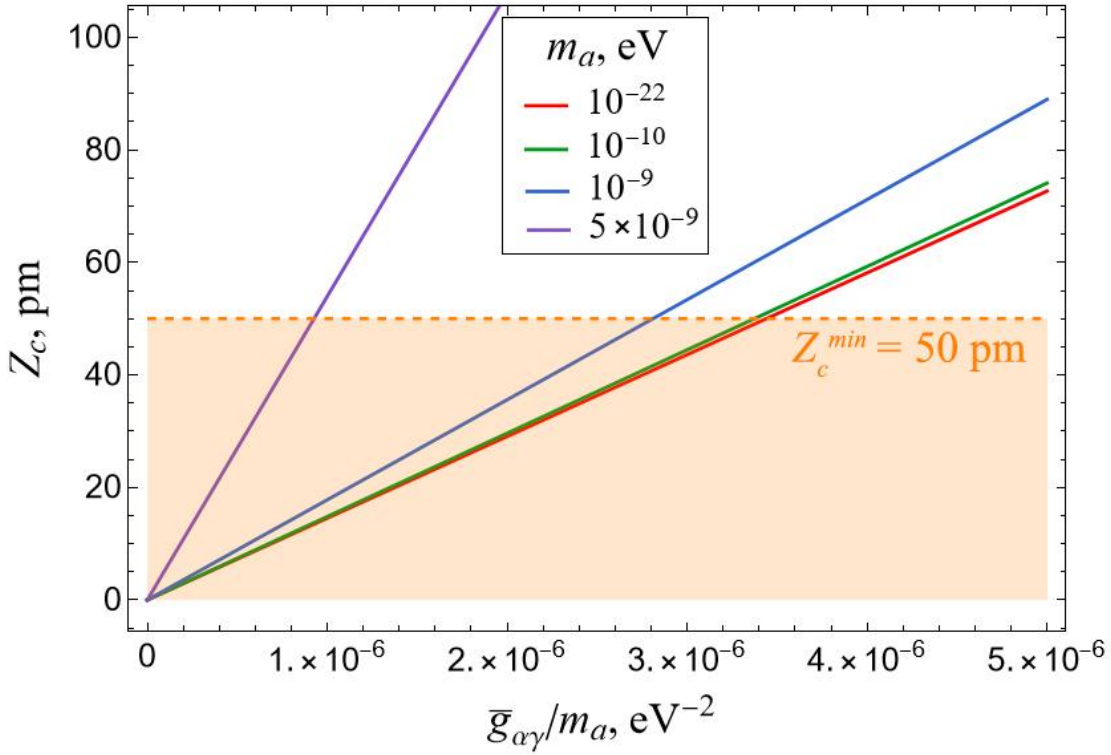


$$\hat{U}(t)\left|\psi\right\rangle_N = \frac{1}{\sqrt{2}}\left|\uparrow\right\rangle_N + e^{i\theta(t)}\frac{1}{\sqrt{2}}\left|\downarrow\right\rangle_N$$

$$\theta(t) = \underbrace{F_0/\hbar \cdot Z_c \tau}_{\theta_{max}} \cos\delta$$

Force sensing

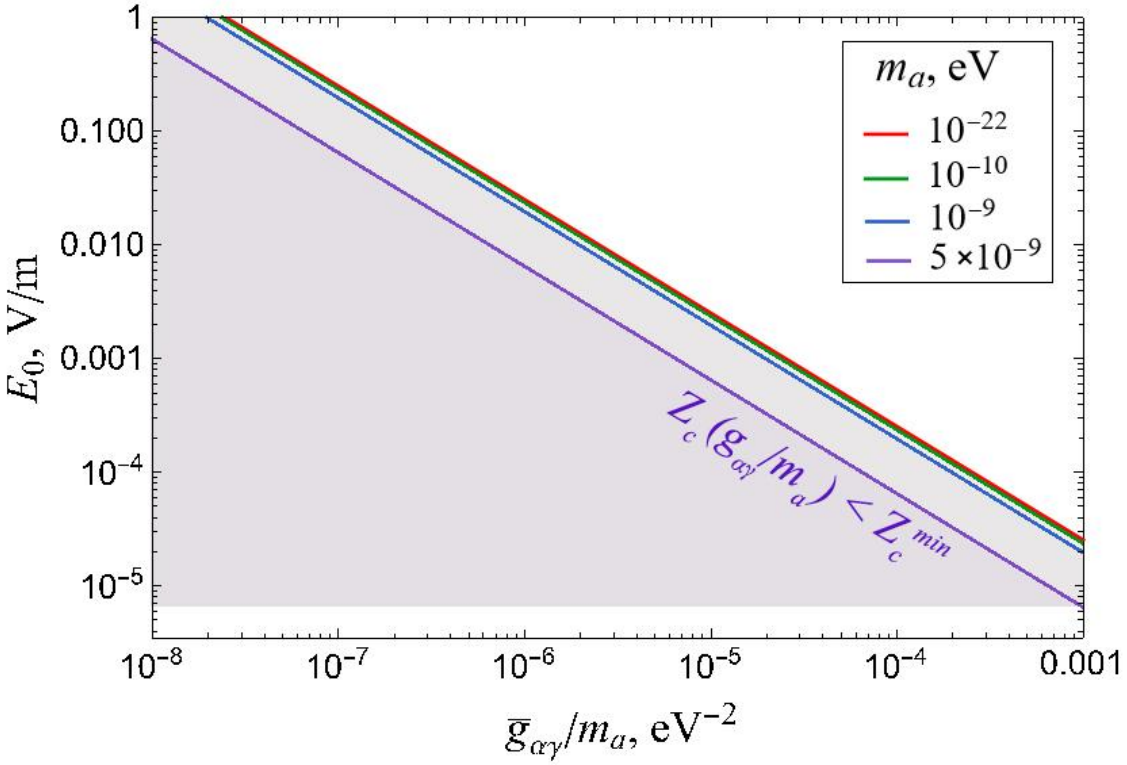
Protocol



$$Z_c = \frac{Z e_{em} E_0}{4M} \frac{\sqrt{2\rho_{DM}}\bar{g}_{\alpha\gamma}}{(\omega - m_a)^2 m_a}$$

Force sensing

Protocol



$$Z_c = \frac{Z e_{em} E_0}{4M} \frac{\sqrt{2\rho_{DM}}\bar{g}_{\alpha\gamma}}{(\omega - m_a)^2 m_a}$$

SNR: $$\frac{Z_c^2}{\delta Z_c^2} \approx \left[\frac{Z_c}{0.2\ \text{nm}}\right]^2$$

Force sensing

Pro's and con's

Pro's

- Simple Ramsey-type experiment
- SNR can be improved with increasing N and reducing Γ

Con's

- For smaller values of axion parameter need stronger EM fields
- Not sensitive enough

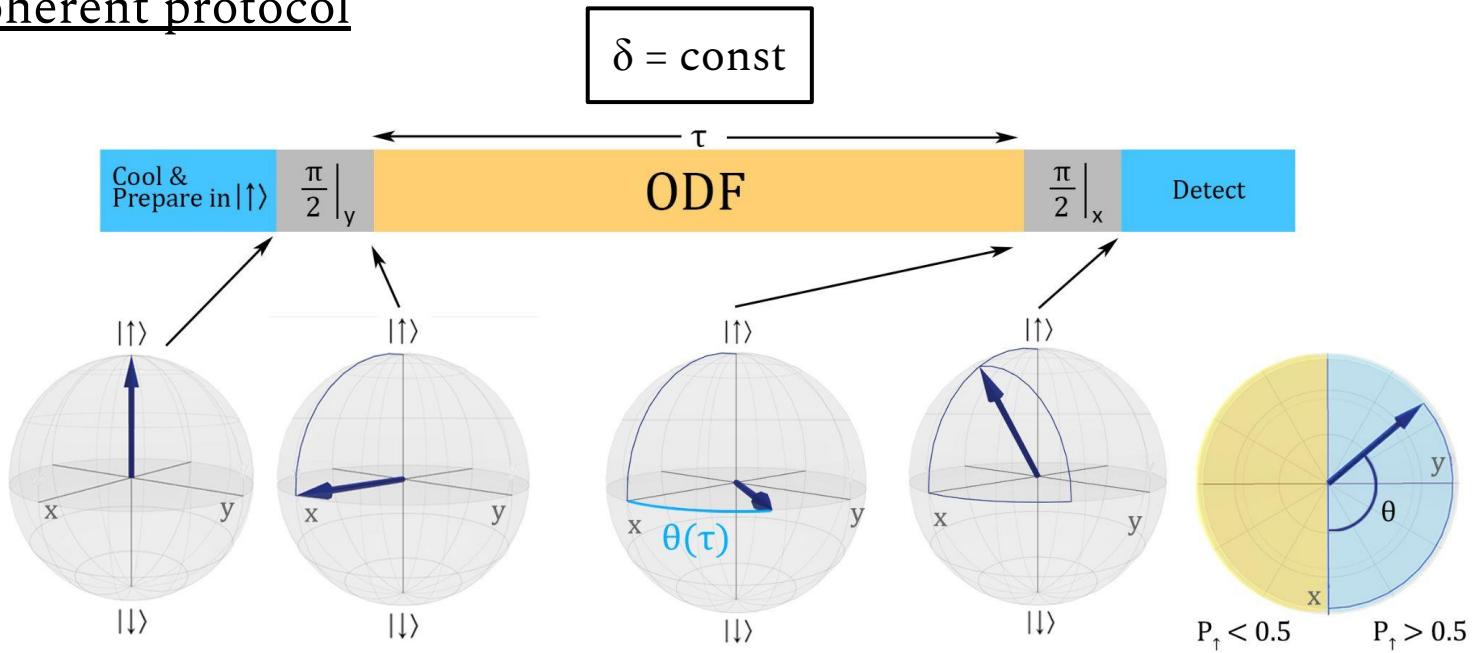


Image: SmarterEveryDay on youtube

Can be improved!

Force sensing

Phase-coherent protocol



$$Z_c^{\min} = 20 \text{ pm} \qquad \frac{Z_c}{\delta Z_c} \approx \frac{Z_c}{0.074 \text{ nm}}$$

Force sensing

Closer to COM frequency

Sensing COM motion resonant with ω_z is much more sensitive.

But! Thermal noise
Zero-point noise
Entanglement-induced back action

Force sensing

Closer to COM frequency

Sensing COM motion resonant with ω_z is much more sensitive.

But! Thermal noise
Zero-point noise
Entanglement-induced back action

Time reversal technique

+spin squeezing

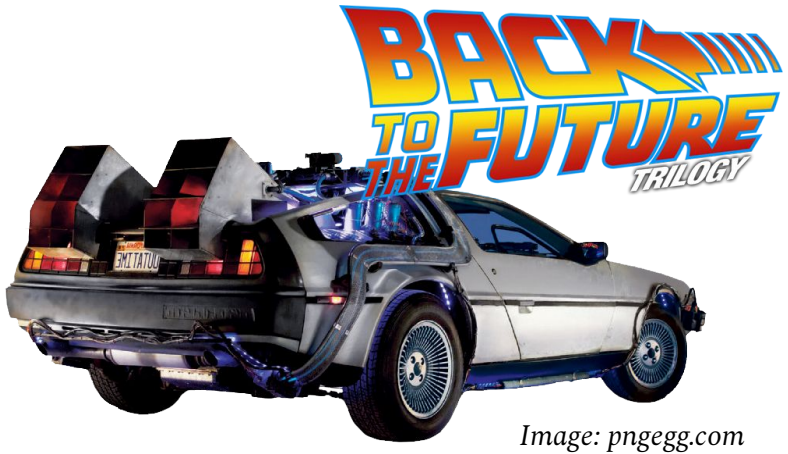


Image: pngegg.com

Force sensing

At the COM frequency

Quantum-enhanced sensing of displacements and electric fields with large trapped-ion crystals

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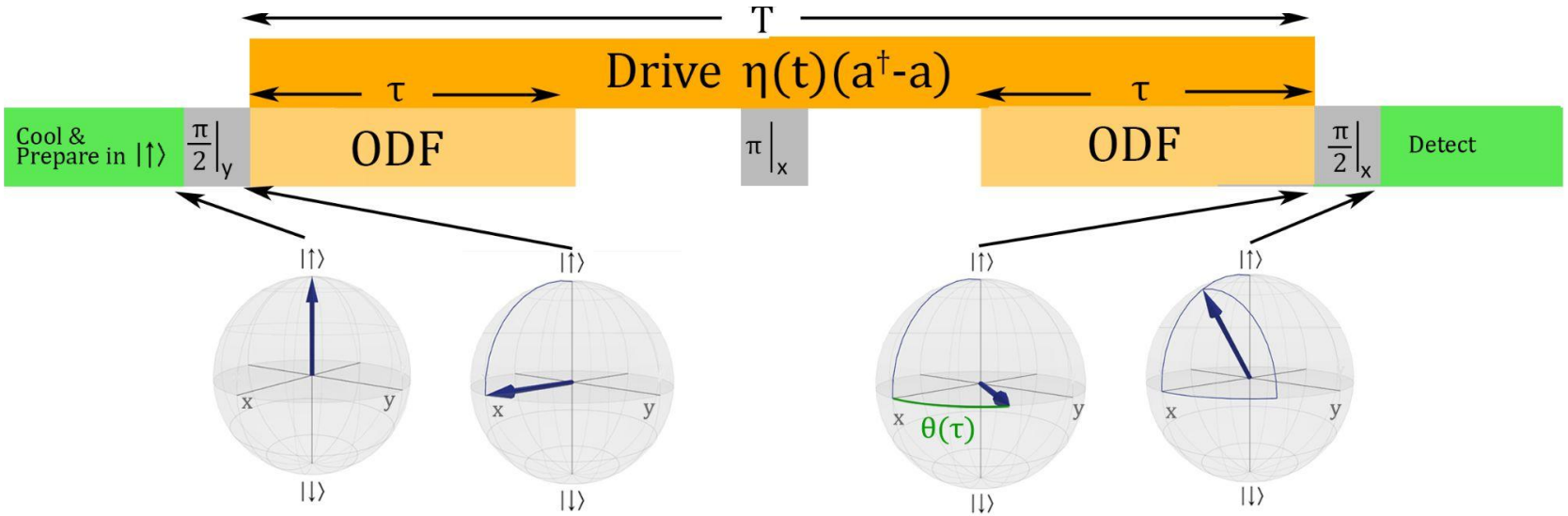
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Developing the isolation and control of ultracold atomic systems to the level of single quanta has led to significant advances in quantum sensing, yet demonstrating a quantum advantage in real world applications by harnessing entanglement remains a core task. Here, we realize a many-body quantum-enhanced sensor to detect weak displacements and electric fields using a large crystal of ~ 150 trapped ions. The center of mass vibrational mode of the crystal serves as high-Q mechanical oscillator and the collective electronic spin as the measurement device. By entangling the oscillator and the collective spin before the displacement is applied and by controlling the coherent dynamics via a many-body echo we are able to utilize the delicate spin-motion entanglement to map the displacement into a spin rotation such that we avoid quantum back-action and cancel detrimental thermal noise. We report quantum enhanced sensitivity to displacements of 8.8 ± 0.4 dB below the standard quantum limit and a sensitivity for measuring electric fields of 240 ± 10 nVm^{-1} in 1 second (240 nVm^{-1}/$\sqrt{\text{Hz}}$).

Force sensing

"Quantum" approach – on the COM frequency

$$\hat{H}_{\text{sens}} = \frac{\hbar g(t)}{\sqrt{N}}(\hat{a} + \hat{a}^{\dagger})\hat{J}_z + i\eta(t)(\hat{a} - \hat{a}^{\dagger})$$

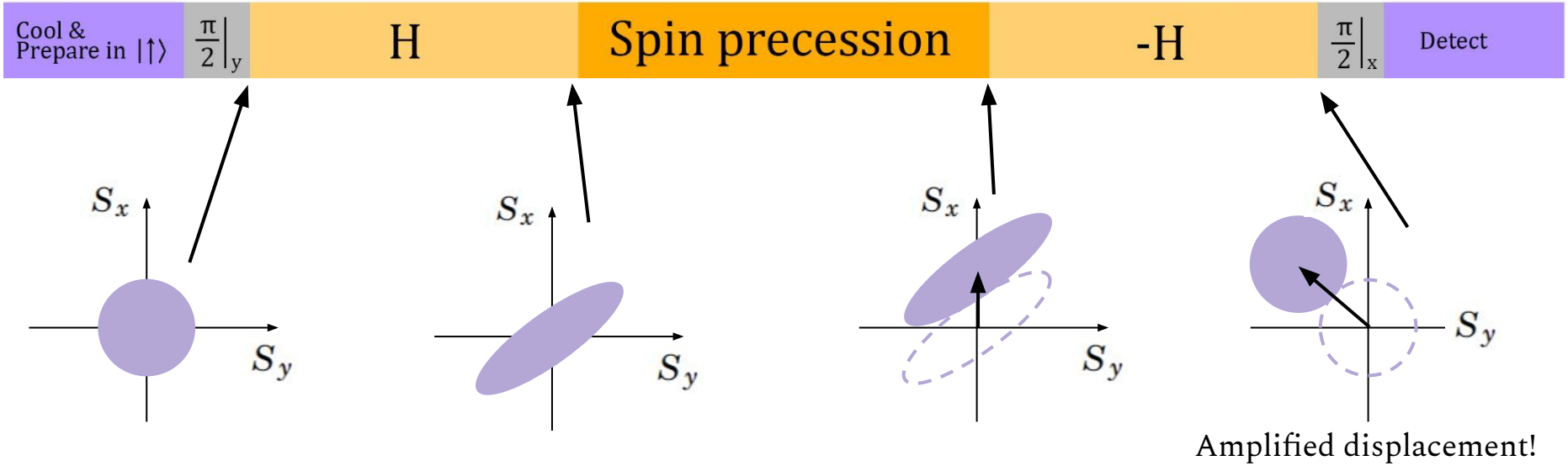


Force sensing

Spin-squeezing



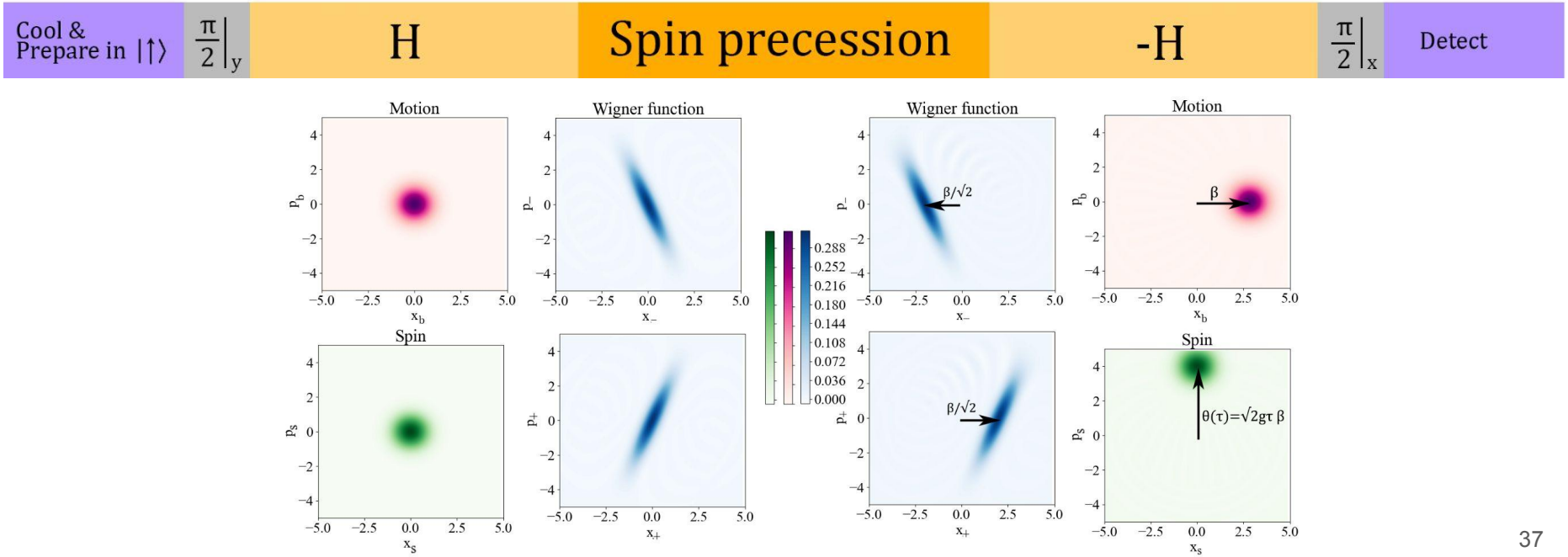
$$\hat{H} = \sum_{i \neq j}^{N} J_{ij} \hat{\sigma}_z^i \hat{\sigma}_z^j = J \hat{S}_z^2$$



Force sensing

Spin-squeezing

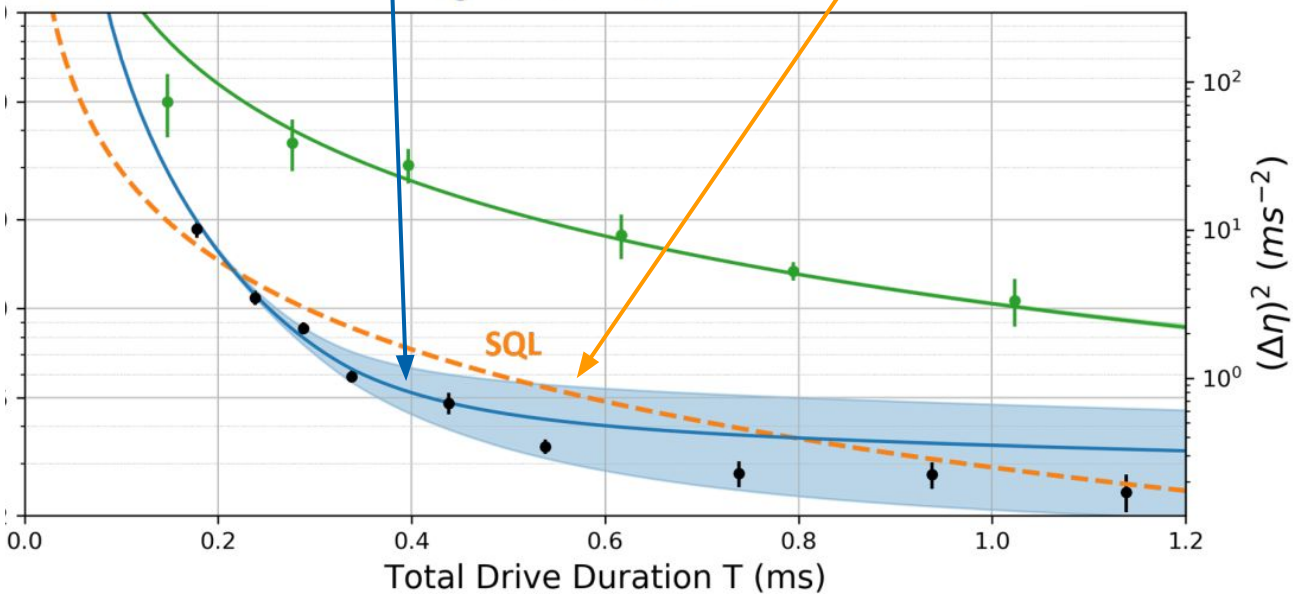
$$\hat{H} = \sum_{i \neq j}^{N} J_{ij} \hat{\sigma}_z^i \hat{\sigma}_z^j = J \hat{S}_z^2$$



Force sensing

"Quantum" approach

$$(\Delta\eta)^2 = \frac{1}{4g^2\tau^2(T-\tau)^2} \qquad (\Delta\eta)^2_{SQL} = \frac{1}{4T^2}$$



Sensitivities:

$$\Delta Z_c^{\text{quantum}} \approx 1.7\ \text{pm}/\sqrt{\text{Hz}}$$

$$\Delta Z_c^{\delta=\text{const}} \approx 18\ \text{pm}/\sqrt{\text{Hz}}$$

$$\Delta Z_c^{\delta\neq\text{const}} \approx (100\ \text{pm})^2/\sqrt{\text{Hz}}$$

Image: "Quantum-enhanced sensing of displacements and electric fields with large trapped-ion crystals" K. A. Gilmore et al.

Conclusions

And future work

1. Learnt and derived for myself important theory on ion traps and laser-ion interactions

2. Introduced myself to axions and their electrodynamics

3. Investigated various methods in measuring axion parameters:
   Classical approaches - phase-incoherent and phase-coherent

"Quantum" approach, quantum metrology and QFI

Thank you for your attention!

Any questions?