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Institute for Theoretical Physics

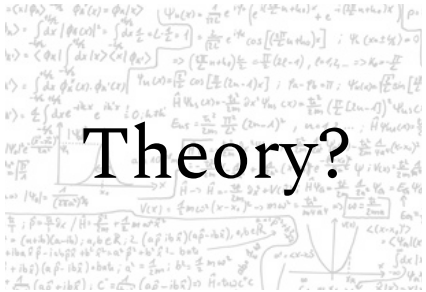
Measuring Oscillating Axion Fields Using Paul Traps & Quantum trajectories

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Introduction

Strong CP problem?



Theory?

AXIONS?

Dark Matter?

Sensing?

Introduction

Axions exist

⌄

Their field interacts with ions

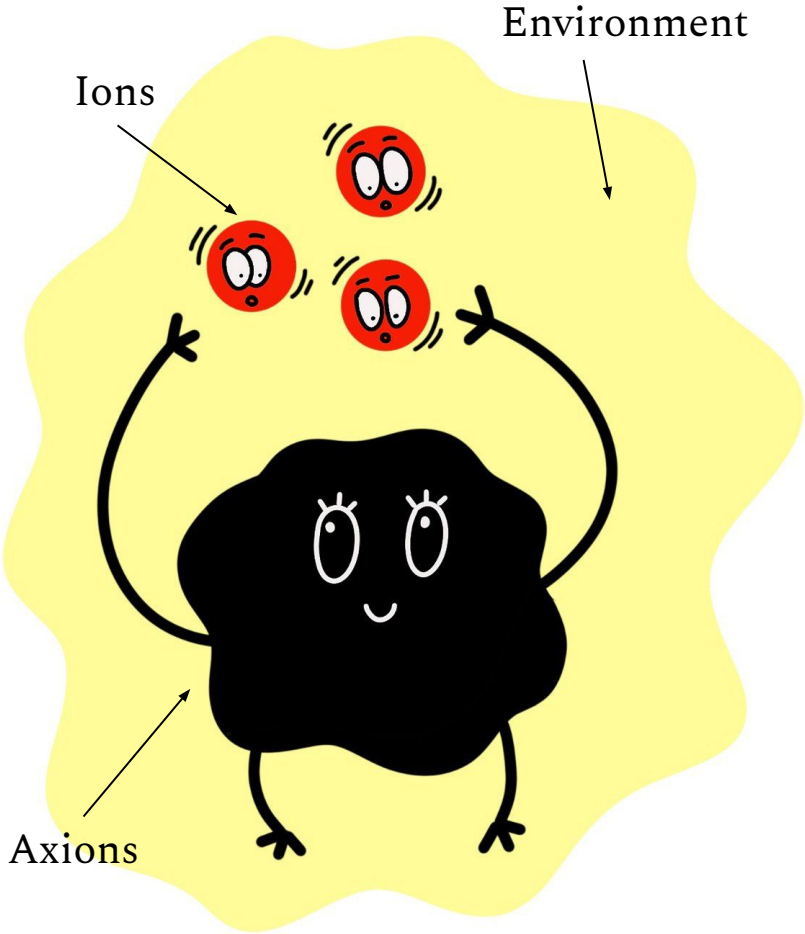
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We measure this interaction

⌄

We know parameters of axions

(hopefully)



Axion electrodynamics

$$L_{QCD} = -\frac{1}{4}\underbrace{G^a_{\mu\nu}G^{\mu\nu}_a}_{CP\checkmark} + \frac{\alpha_s}{8\pi}\bar{\theta}\underbrace{G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a}_{CP\times} + \underbrace{\bar{\psi}_i(i\gamma^\mu D_\mu - M)\psi_i}_{CP\checkmark\times 2}$$

- Addition of an axion field that at long times axion field absorbs theta term

But! New particle axion, which has not been yet discovered experimentally

$$m_a \approx 6\mu eV\left(\frac{10^{12}GeV}{f_a}\right)$$

Dark Matter candidate?



Image: amazon.com

Axion electrodynamics

Time-oscillating e

Consequence: time-oscillating values for EM constants

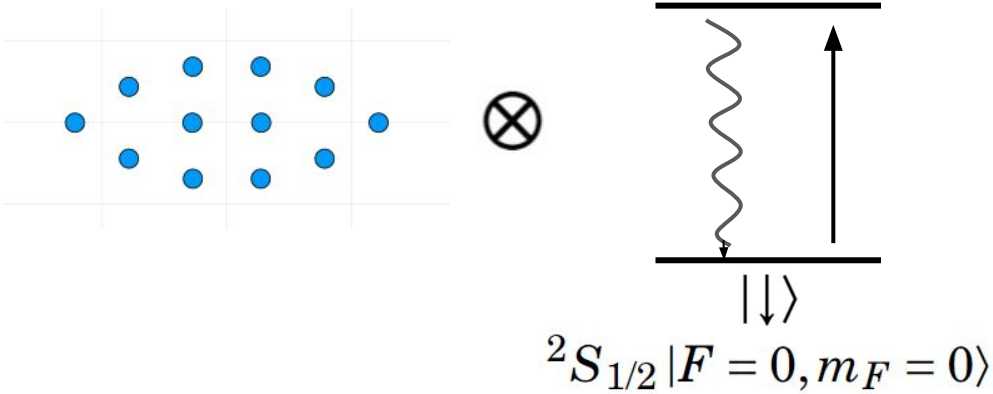
$$\alpha(t) \approx \alpha_{em}(1 + \bar{g}_{a\gamma} a(t)) \Rightarrow e \approx \underbrace{\sqrt{4\pi\epsilon_0 \hbar c \cdot \alpha_{em}}}_{e_{em}} \sqrt{1 + \bar{g}_{a\gamma} a_0 \cos(m_a t)}$$

axion-photon coupling constant

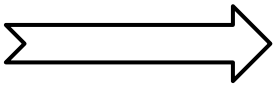
axion mass

Sensing with ions

Couple motion and spin states!



Axion field effects the motion of ions



The number of ions in the "up" state is changed!

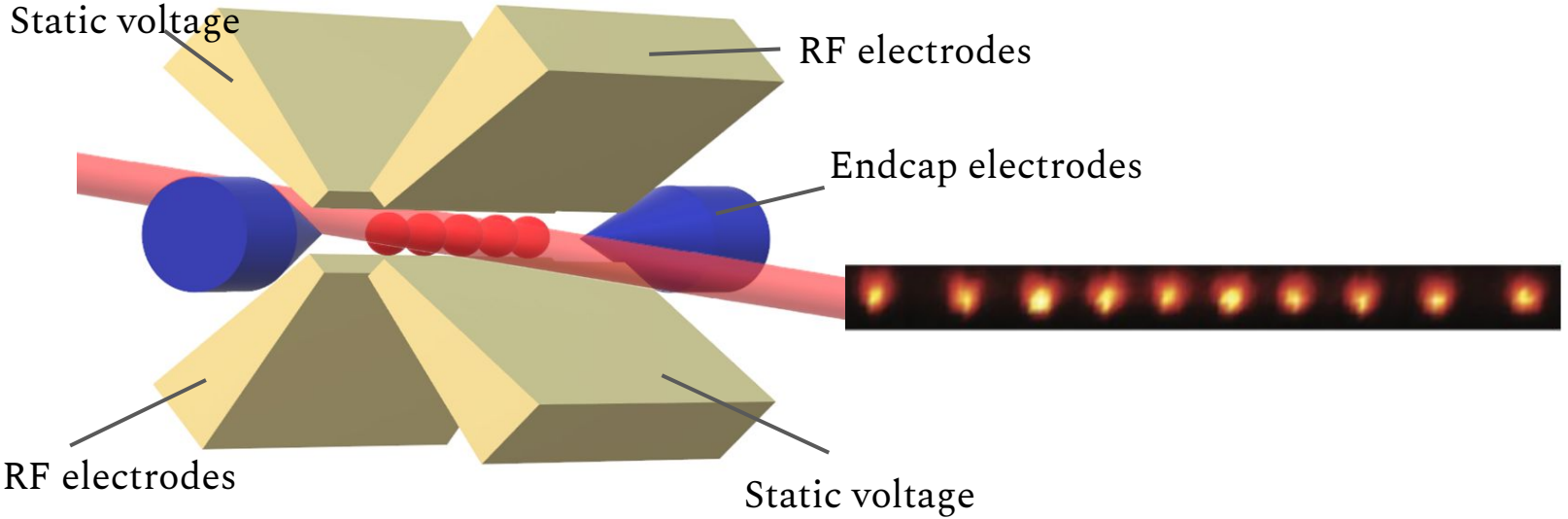
Ion crystals

Trap potential

Paul traps:

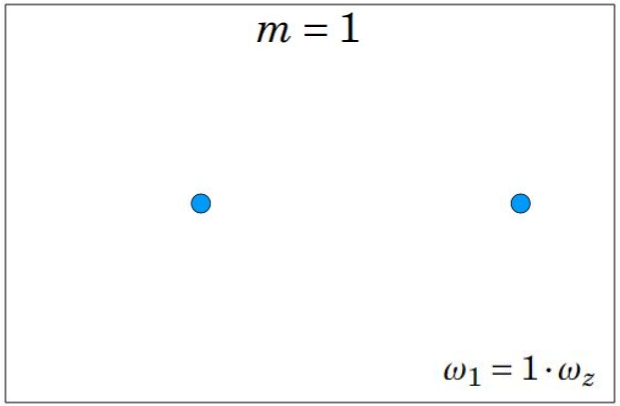
- Electrostatic + oscillating electric fields + Coulomb repulsion

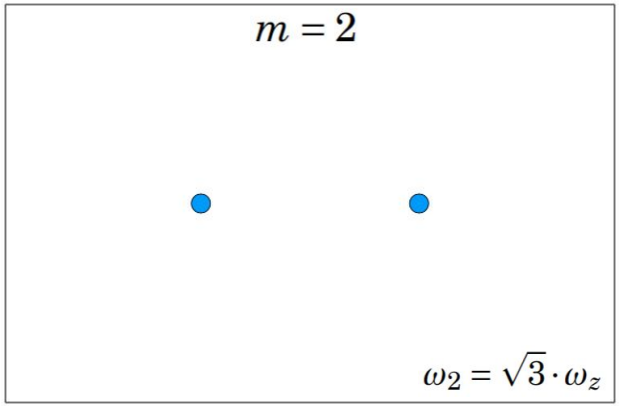
$$V = \sum_{i=1}^{N} \frac{1}{2} M(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2) + \sum_{i,j=1, j\neq i}^{N} \frac{Z^2 e^2}{8\pi\epsilon_0} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$



Ion crystals

Motional modes





Center-of-mass (COM) mode

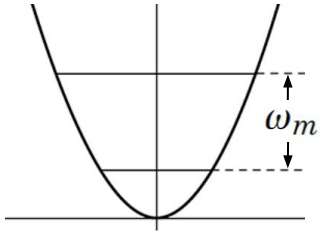
$$\mathbf{b}_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

Breathing mode

$$\mathbf{b}_2 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

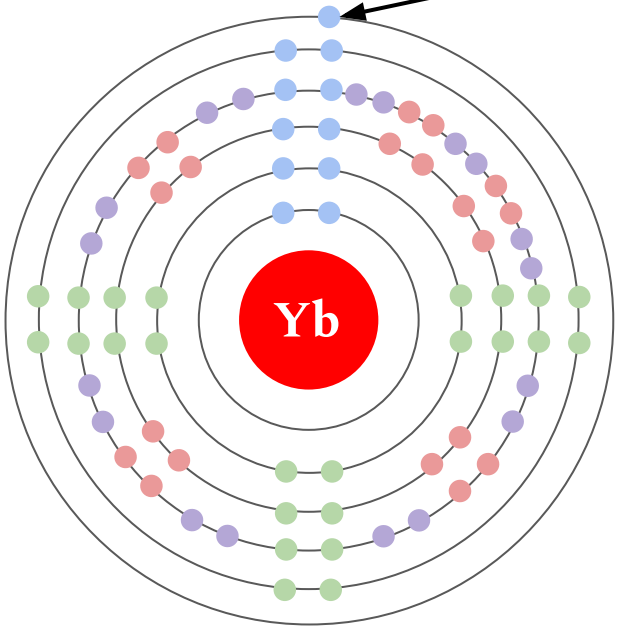
Hamiltonian in terms of motional modes:

$$\hat{H}_{\text{motion}} = \sum_{m=1}^{3N} \hbar\omega_m \left(\hat{a}_m^\dagger \hat{a}_m + \frac{1}{2}\right)$$



Spin dependent forces

^{171}Yb+



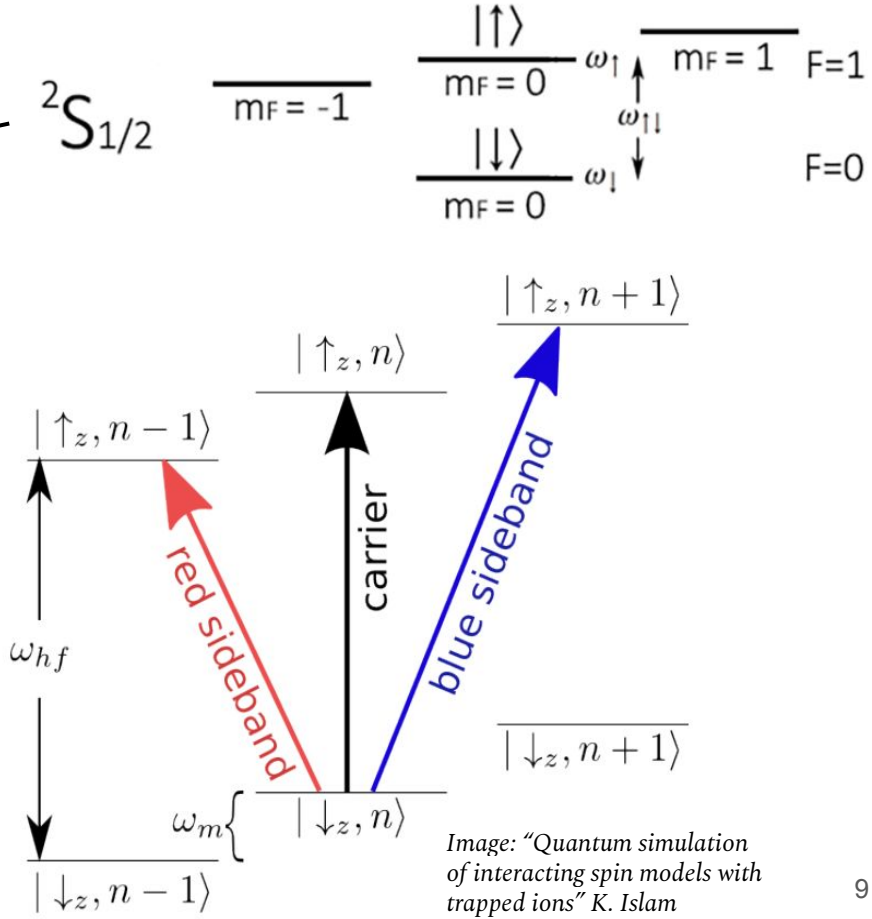


Image: "Quantum simulation of interacting spin models with trapped ions" K. Islam

Force sensing

Optical-dipole force

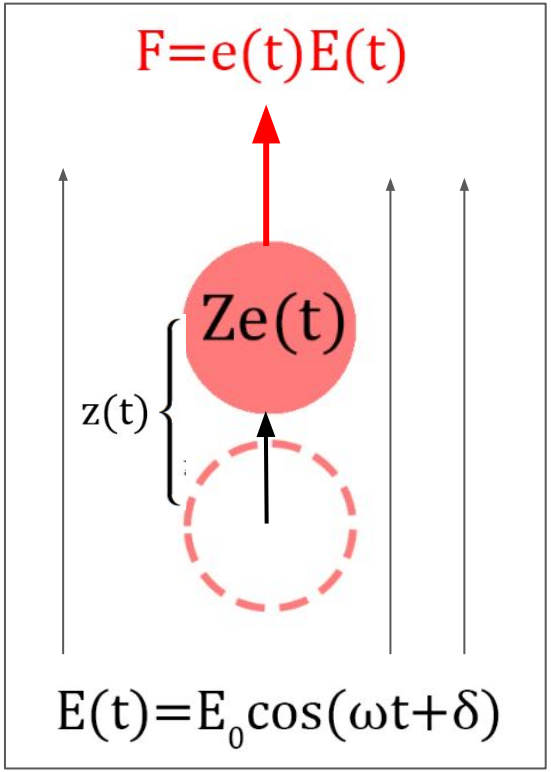
$$\hat{H}_{\mathrm{ODF}} = \sum_{i=1}^{N} F_0 \cos(\mu t)\hat{z}_i \hat{\sigma}_z^i$$

Apply external electric field, so that $\hat{z}_i \rightarrow \hat{z}_i + z(t)$

$$\hat{H}_{ODF} \approx F_0 \frac{Z e_{em} E_0}{4M} \frac{\bar{g}_{\alpha\gamma} a_0}{(\omega - m_a)^2} \left(\cos\left((\omega - m_a - \mu)t + \delta\right)\right) \sum_{i=1}^{N} \frac{\hat{\sigma}_z^i}{2}$$

Amplitude of displacement:

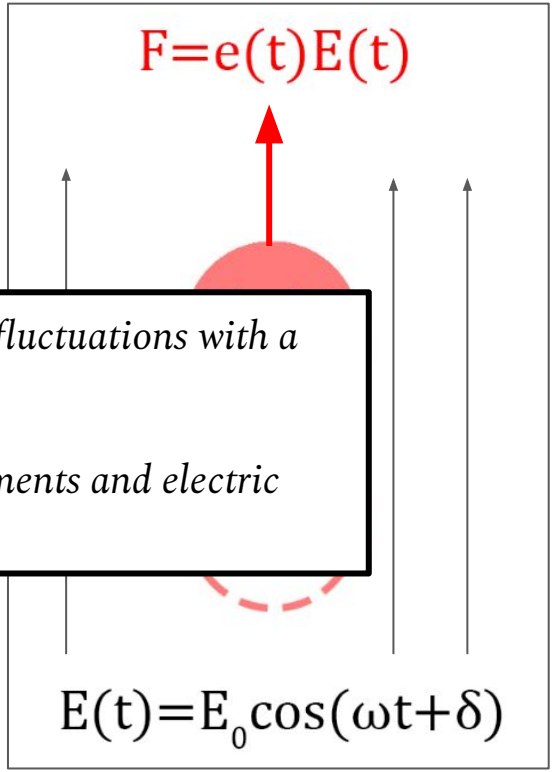
$$Z_c = \frac{Z e_{em} E_0}{4M} \frac{\sqrt{2\rho_{DM}}\bar{g}_{\alpha\gamma}}{(\omega - m_a)^2 m_a}$$



Force sensing

Optical-dipole force

$$\hat{H}_{\mathrm{ODF}} = \sum_{i=1}^{N} F_0 \cos(\mu t)\hat{z}_i \hat{\sigma}_z^i$$



K. A. Gilmore et al., "Amplitude sensing below the zero-point fluctuations with a two-dimensional trapped-ion mechanical oscillator"

K. A. Gilmore et al., "Quantum-enhanced sensing of displacements and electric fields with two-dimensional trapped-ion crystals"

Apply ext

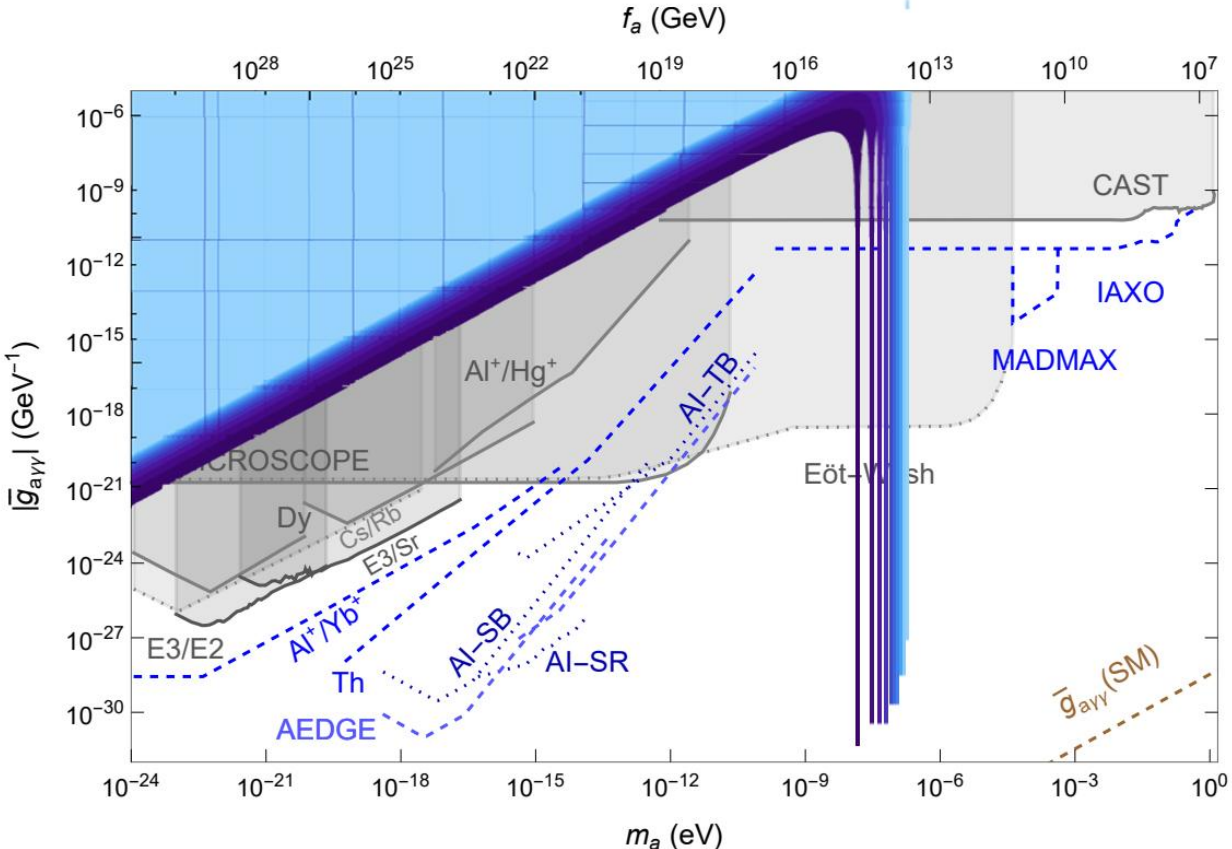
$$\hat{H}_{ODF} \approx F \left(\frac{}{4M} \frac{}{(\omega - m_a)^2} \right) \sum_{i=1} \frac{}{2}$$

Amplitude of displacement:

$$Z_c = \frac{Z e_{em} E_0}{4M} \frac{\sqrt{2\rho_{DM}}\bar{g}_{\alpha\gamma}}{(\omega - m_a)^2 m_a}$$

Force sensing

Parameter space



$$Z_c = 50 \text{ pm}$$

$$\omega/2\pi = 100 \text{ kHz}$$

$$\omega/2\pi = 1000 \text{ kHz}$$

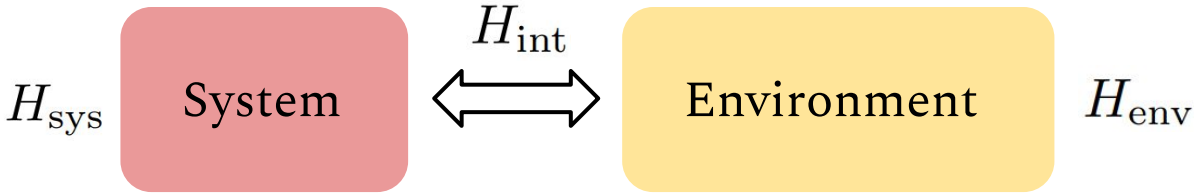
$$Z_c = \frac{Z e_{em} E_0}{4M} \frac{\sqrt{2\rho_{DM}}\bar{g}_{\alpha\gamma}}{(\omega - m_a)^2 m_a}$$

Improvements

With master equation

Master equation is a very powerful method to do:

1. Noise Characterization
2. Dynamics of Quantum System
3. Quantum Parameter Estimation and others



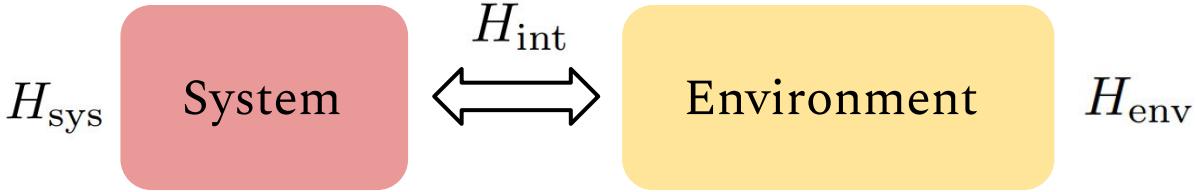
$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H,\rho] - \frac{1}{2}\sum_m \gamma_m \left[\tilde{c}_m^\dagger \tilde{c}_m \rho + \rho \tilde{c}_m^\dagger \tilde{c}_m - 2\tilde{c}_m \rho \tilde{c}_m^\dagger\right]$$

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The state of the system
$\rho = \mathrm{Tr}_{\text{env}}\{\rho_{\text{total}}\}$

Hamiltonian evolution

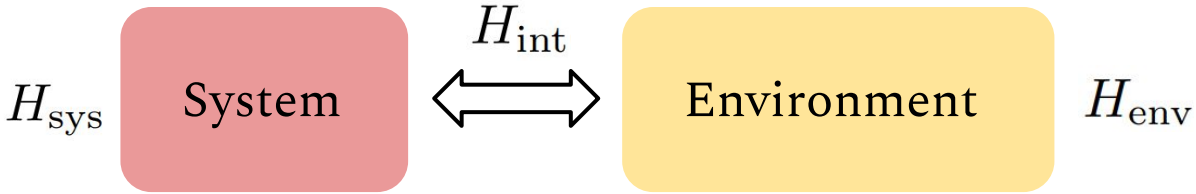
Lindbladian superoperator describing decoherence/loss

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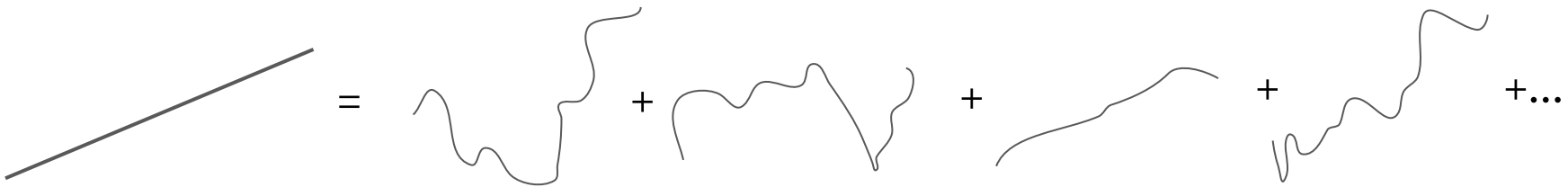
Directly integrate

Use other methods

Improvements

Quantum trajectory techniques

time evolution of the quantum system = $\sum$ individual stochastic trajectories that account for both unitary evolution and quantum jumps due to dissipation



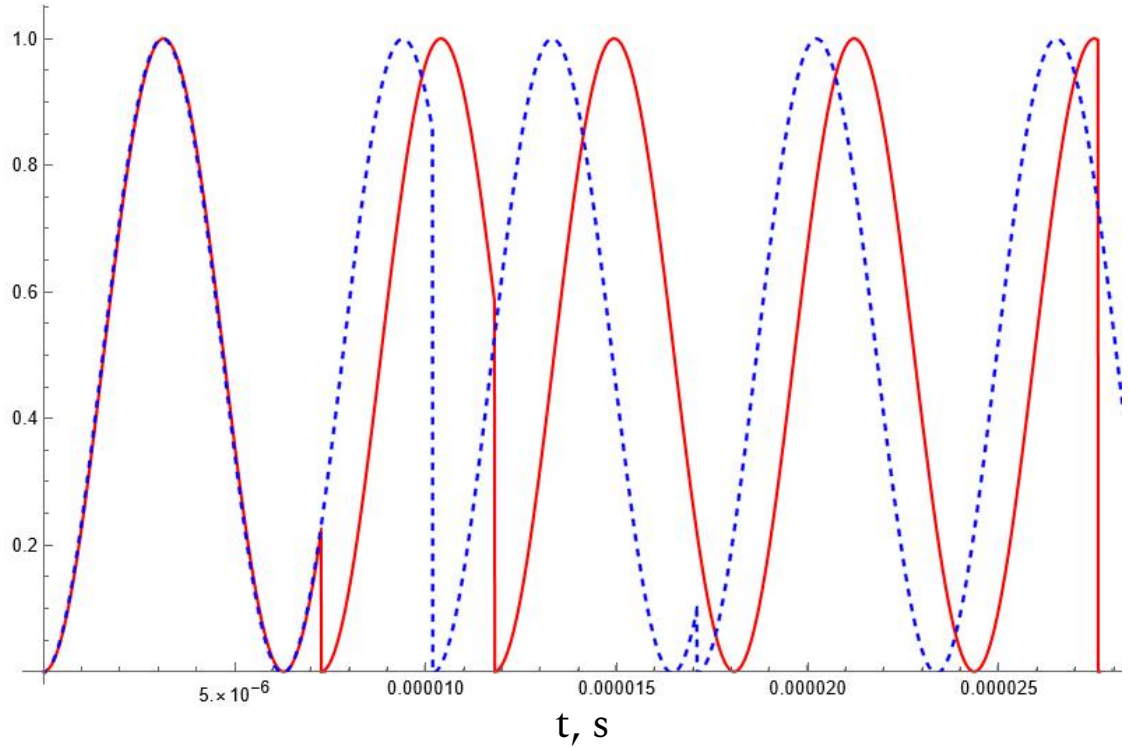
Decoherence

Illustrative example: Optical Bloch Equations

$$\frac{d}{dt}\rho = -\mathrm{i}[H_{\mathrm{opt}}, \rho] - \frac{\Gamma}{2}\left(\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_- - 2\sigma_-\rho\sigma_+\right)$$

$$H_{\mathrm{opt}} = -\frac{\Omega}{2}\sigma_x - \Delta\sigma_+\sigma_-$$

$$c \equiv \sigma_-$$



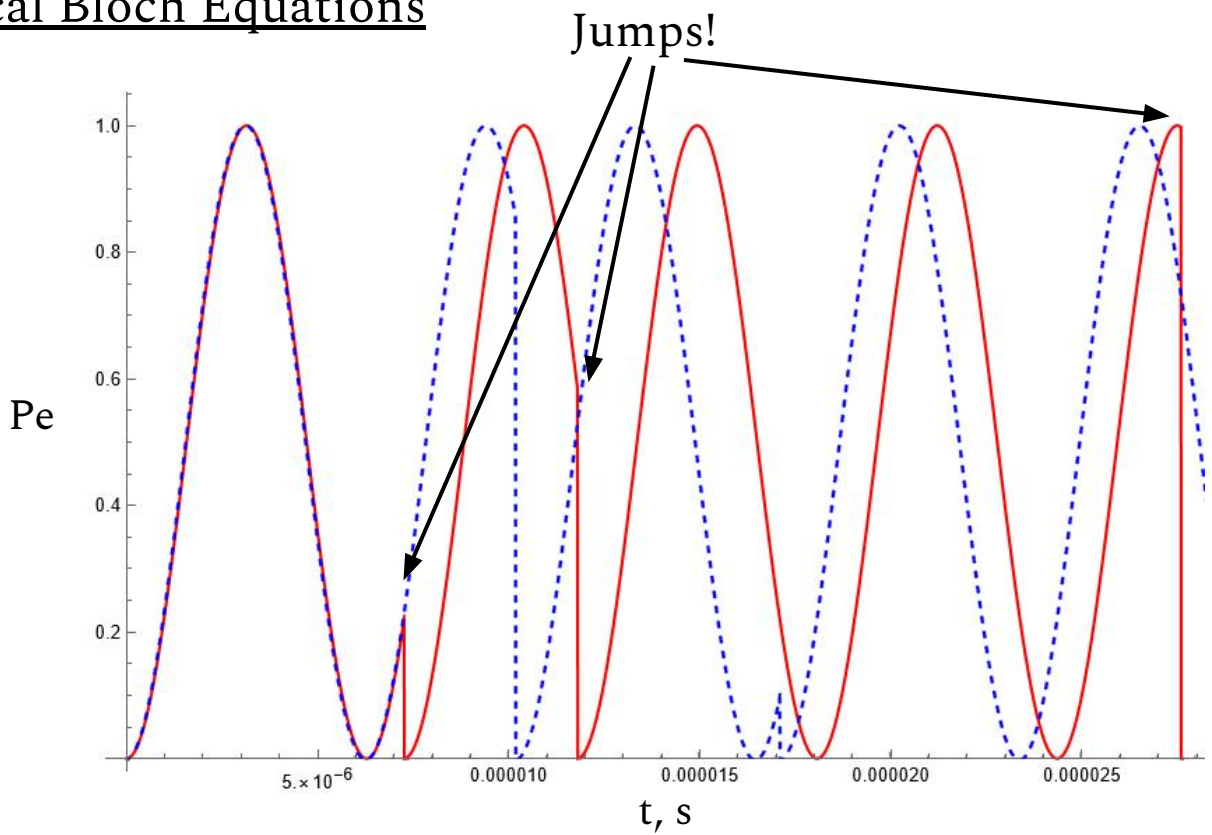
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$$H_{\mathrm{opt}} = -\frac{\Omega}{2}\sigma_x - \Delta\sigma_+\sigma_-$$

$$c \equiv \sigma_-$$



Decoherence

Illustrative example: Optical Bloch Equations

Clear damping
of oscillations!

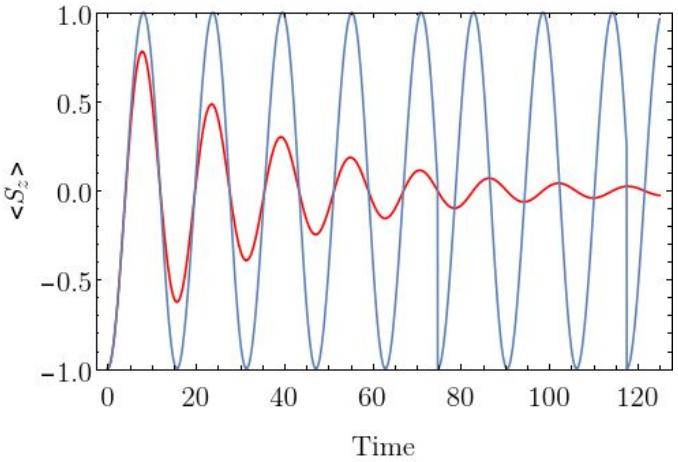
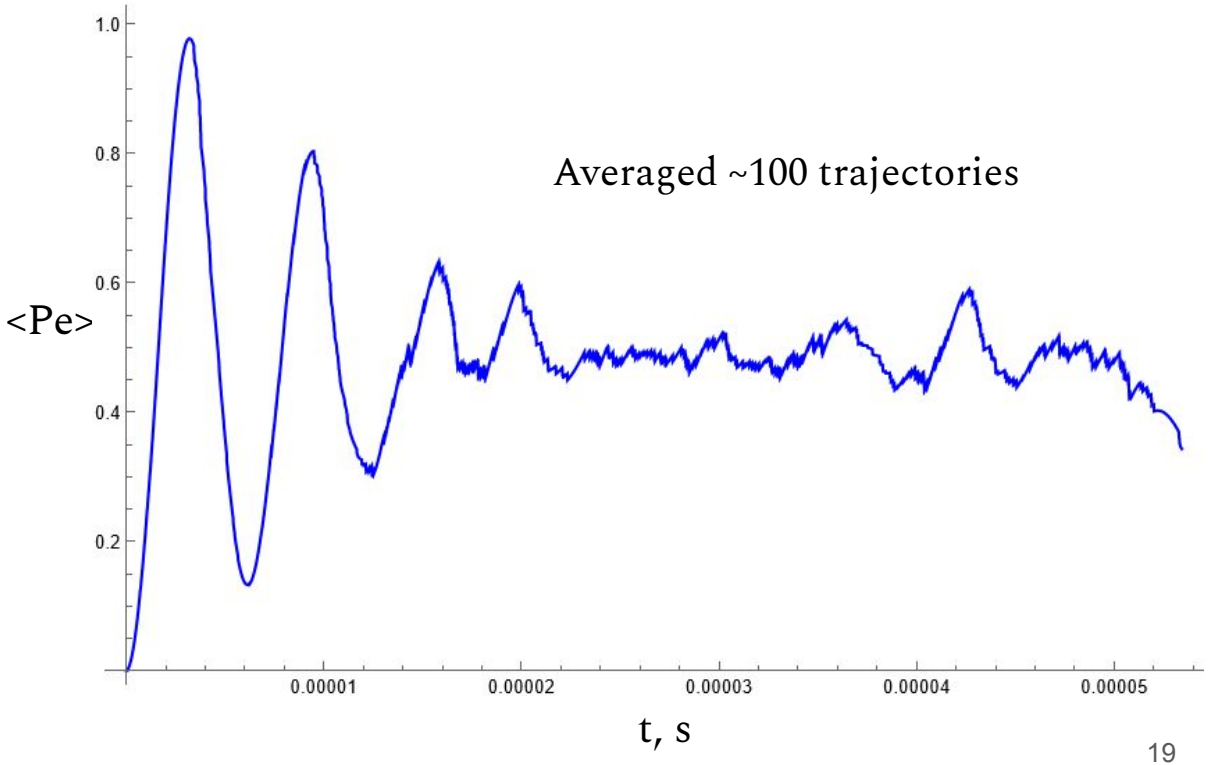


Image: CarlosSMWolff



Averaged ~100 trajectories

Conclusions

And future work

- Derived axion-caused ion motion amplitude.
- Explored master equation and quantum trajectories.

In the future:

Look into the ways to use the master equation and quantum jumps to increase the *sensitivity* and create *better states* and *systems* of the axion sensing experiments.

Thank you for your attention!

Any questions?