(Calculating)

Fully analytical q_T spectrum of radiative corrections

to the Drell-Yan helicity cross sections up to $O(\alpha^2)$

Daria Trotsenko

Supervisors: A.Gavardi, B.Dehnadi, F.Tackman



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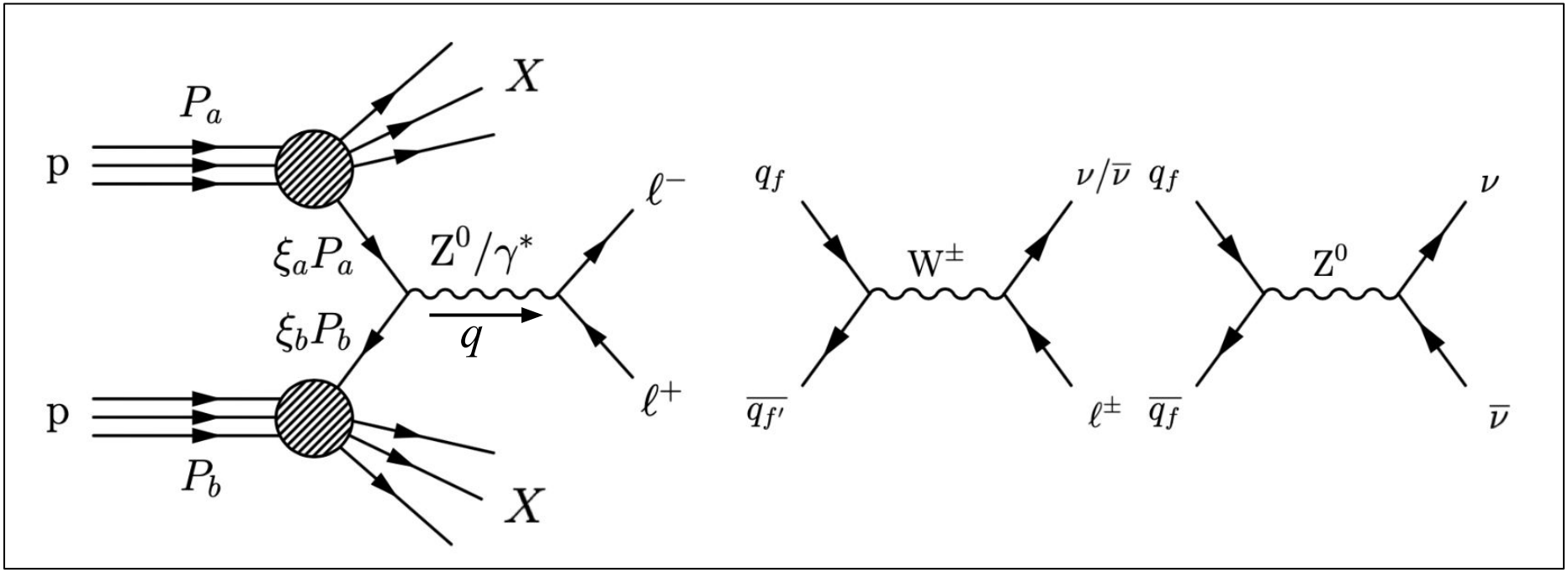
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4. Results:
   a. T-channel interference
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5. Conclusions



My office! :)

Goal of the project

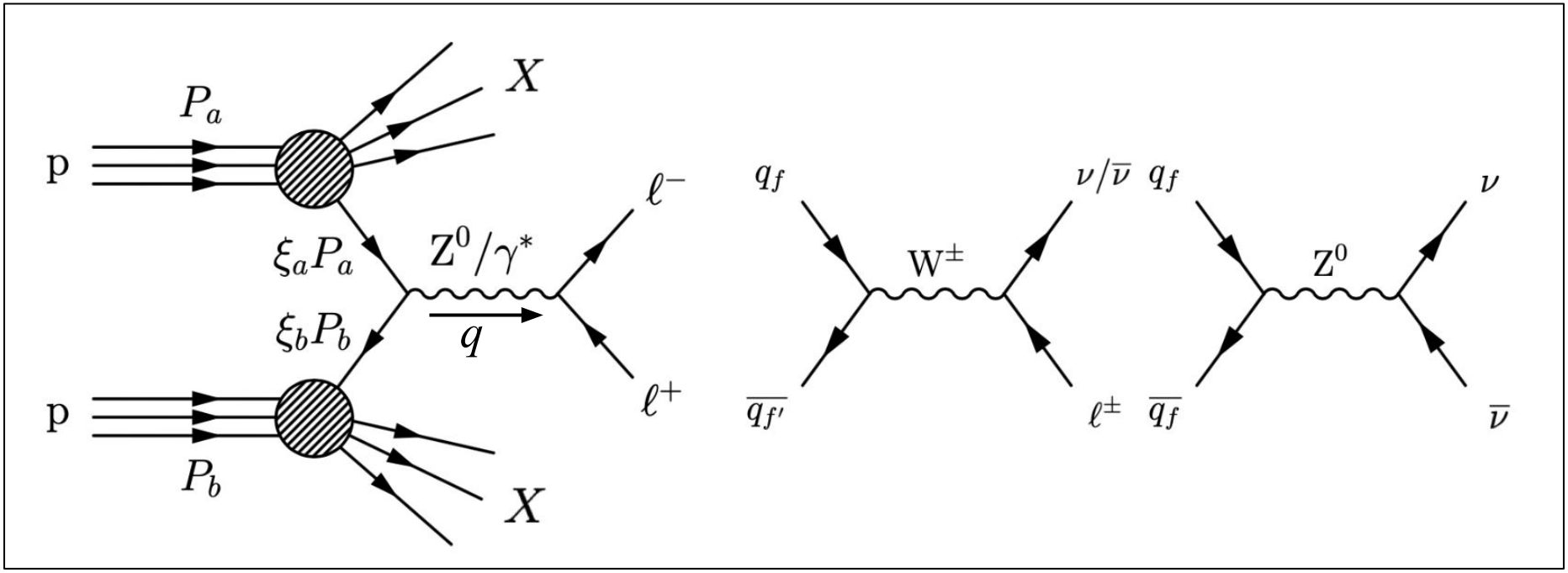
Calculate helicity cross sections for the Drell-Yan process



Drell-Yan process with various intermediate vector bosons

Goal of the project

Calculate helicity cross sections for the Drell-Yan process



Drell-Yan process with various intermediate vector bosons

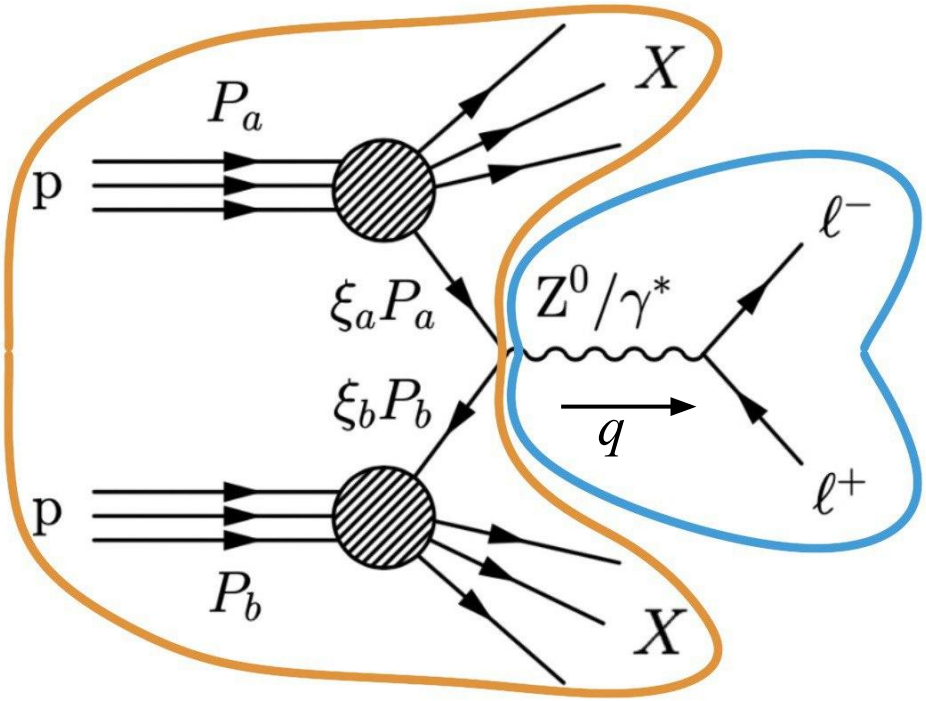
Excellent benchmark for testing the theoretical predictions of the Standard Model

General theory

Differential cross section written in terms of

hadronic and leptonic tensors:

$$\frac{d\sigma(\Theta)}{d^4q\,d\mathcal{O}} = \frac{1}{2E_{\mathrm{cm}}^2} L_{\mu\nu}(q, \mathcal{O}, \Theta) W^{\mu\nu}(q, P_a, P_b)$$



General theory

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$$W^{\mu\nu} = \sum_{i=-1}^{7} P_i^{\mu\nu} W_i,$$

"Drell-Yan q_T Resummation of Fiducial Power Corrections at N3LL" by M.A.Ebert et al.

arXiv:2006.11382

Written as a combination of
structure functions W_i
and
(inverted) *projectors* $P_i^{\mu\nu}$

General theory

Structure functions:

$$W^{\mu\nu} = \sum_{i=-1}^{7} P_i^{\mu\nu} W_i,$$

are among the most relevant observables of the pp scattering.

They are generally calculated from theory and later constrained by experimental measurements of the cross section.

Our goal: analytical calculation of W_i *and cross section* $d\sigma_i^h$.

Using Mathematica and Feyncalc :)

General theory

Structure functions and projectors:

$$W^{\mu\nu} = \sum_{i=-1}^{7} P_i^{\mu\nu} W_i,$$

Structure functions as Lorentz-scalar projections w.r.t. four orthogonal vectors:

$$\begin{aligned}
W_{-1} &= (x^\mu x^\nu + y^\mu y^\nu) W_{\mu\nu}, \\
W_0 &= 2 z^\mu z^\nu W_{\mu\nu}, \\
W_1 &= -(x^\mu z^\nu + z^\mu x^\nu) W_{\mu\nu}, \\
W_2 &= -2(x^\mu x^\nu - y^\mu y^\nu) W_{\mu\nu}, \\
W_3 &= 2i(y^\mu z^\nu - z^\mu y^\nu) W_{\mu\nu}, \\
W_4 &= 2i(x^\mu y^\nu - y^\mu x^\nu) W_{\mu\nu}, \\
W_5 &= -(x^\mu y^\nu + y^\mu x^\nu) W_{\mu\nu}, \\
W_6 &= -(y^\mu z^\nu + z^\mu y^\nu) W_{\mu\nu}, \\
W_7 &= -2i(x^\mu z^\nu - z^\mu x^\nu) W_{\mu\nu}.
\end{aligned}$$

General theory

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Helicity cross sections:

$$\frac{d\sigma_i^h}{d^D q} = \frac{1}{2E_{cm}^2} W_i(q, P_a, P_b)$$

General theory

Structure functions and projectors:

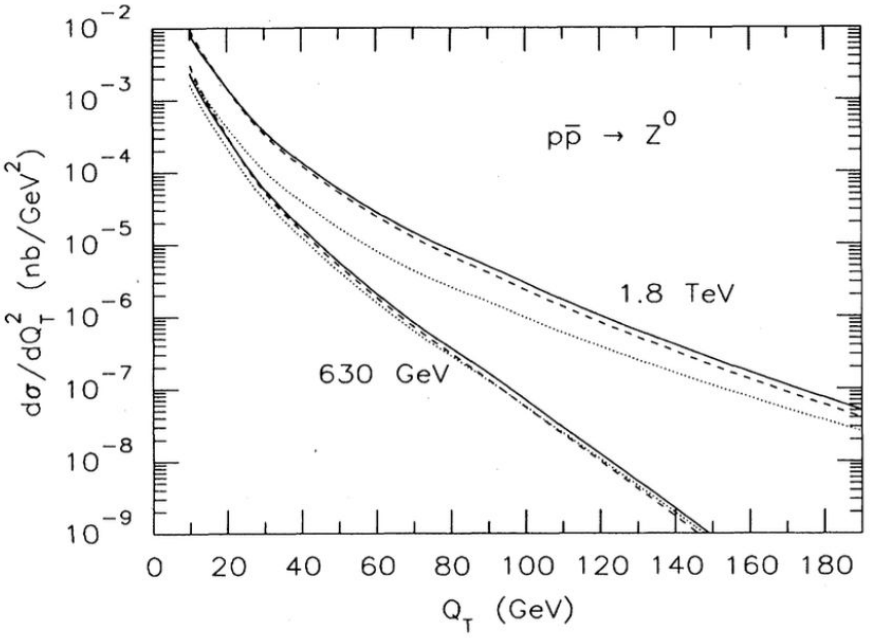
Helicity cross sections:

$$\frac{d\sigma_i^h}{d^D q} = \frac{1}{2E_{cm}^2} W_i(q, P_a, P_b)$$

$$\rightarrow f(q_T^2) \rightarrow$$

R. J. Gonsalves et al.

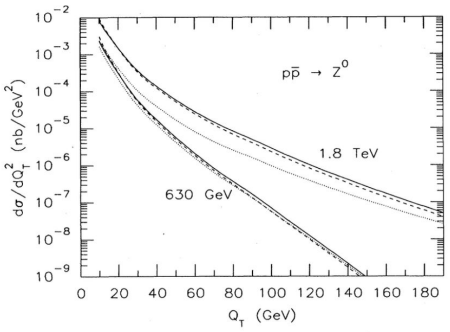
doi.org/10.1103/PhysRevD.40.2245



General theory

Transverse momentum of vector boson

Cross section as a function of transverse momentum



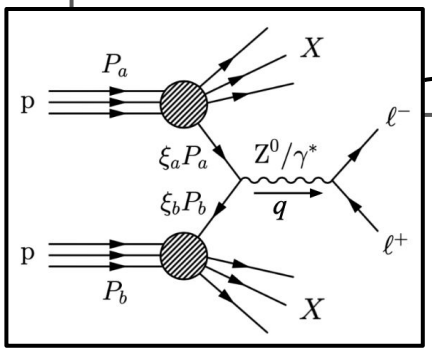
Processes with more strong couplings

Only formulas

(Calculating)

Fully analytical q_T spectrum of radiative corrections

to the Drell-Yan helicity cross sections up to $O(\alpha_s^2)$



Cross section is proportional to α_S^2

Cross sections with structure functions

$$\frac{d\sigma_i}{dq_T^2} = f(W_i)$$

Sorting the diagrams

Real diagrams: no loops

$$q\bar{q} \to q\bar{q}V, \quad q\bar{q} \to ggV, \quad qg \to qgV, \quad qq \to qqV, \quad gg \to q\bar{q}V,$$

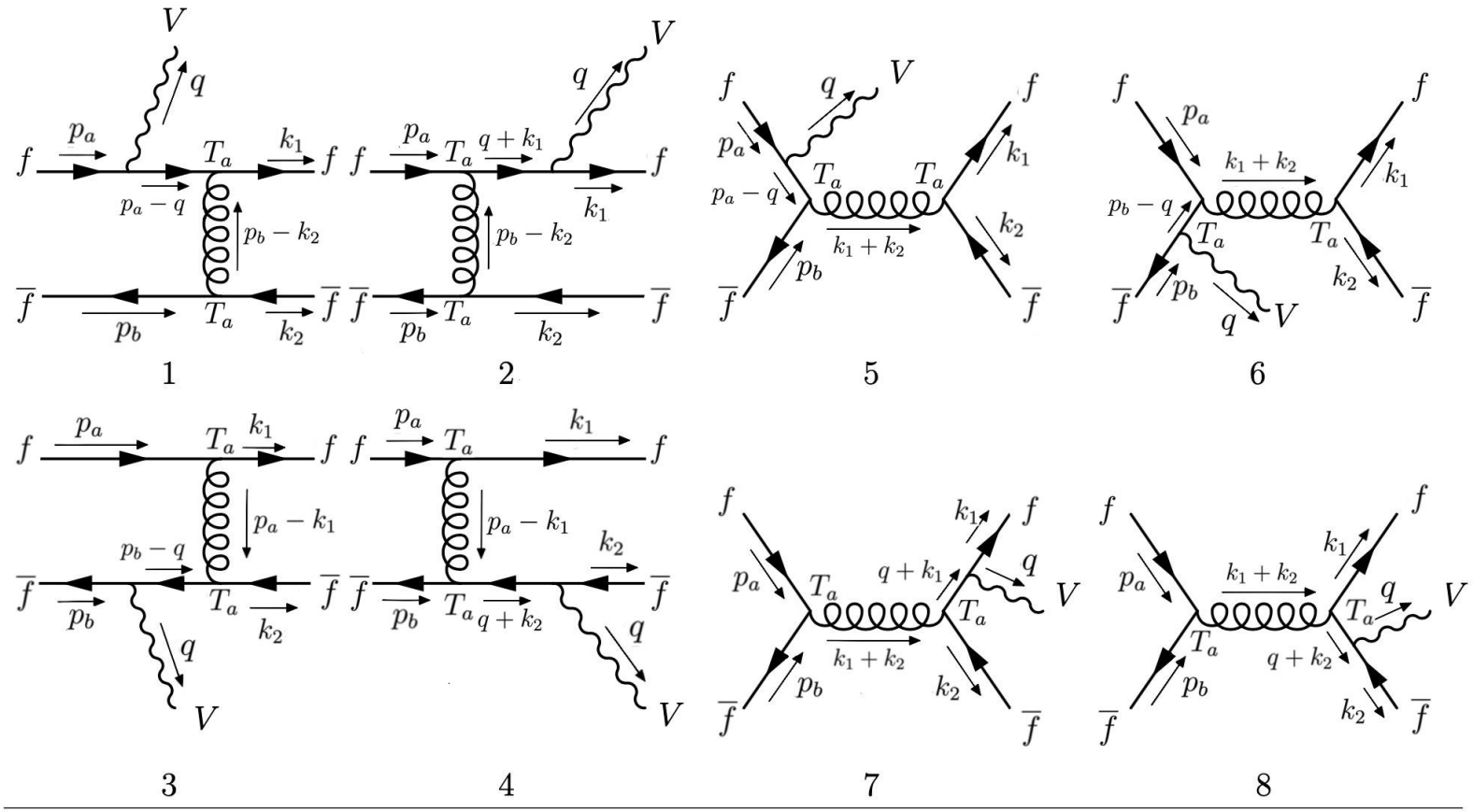
Virtual diagrams: including loops

$$q\bar{q} \to gV, \quad qg \to qV.$$

Real-real (RR) and real-virtual (RV) processes

Sorting the diagrams

Our interest: $q\bar{q} \to q\bar{q}V$



Sorting the diagrams

$$D = 4 - 2\epsilon$$

Pole structure:

RR: only IR poles $\frac{1}{\epsilon_{IR}}$ — **Final states collinear**

RV: both IR and UV $-\frac{1}{\epsilon_{IR}} + \frac{1}{\epsilon_{UV}}$ — **Final states collinear + loops**

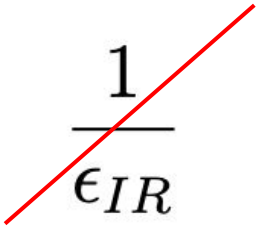
Single R at NNLO: $-\frac{1}{\epsilon_{UV}}$ — α_S **+ PDF renormalization**

Sorting the diagrams

$$D = 4 - 2\epsilon$$

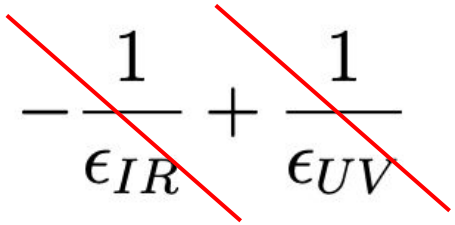
Pole structure: special case

RR: only IR poles



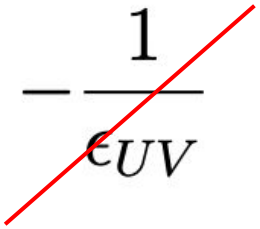
Final states collinear

RV: both IR and UV



Final states collinear + loops

Single R at NNLO:



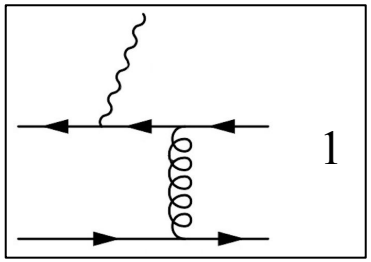
α_S **+ PDF renormalization**

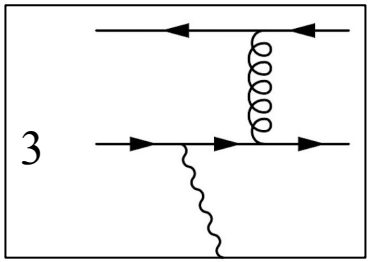
Sorting the diagrams

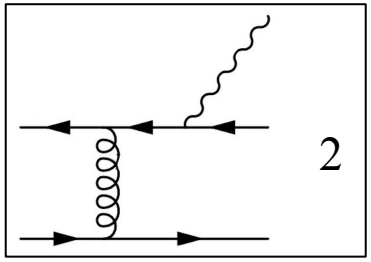
Color and charge structure:

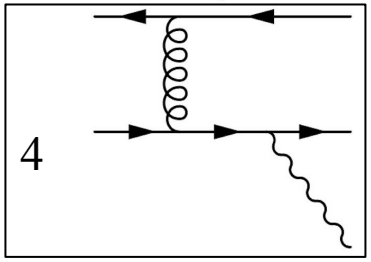
	1	2	3	4	5	6	7	8
1	$T_F C_F N_C$				$C_F\left(C_F - \frac{C_A}{2}\right) N_C$			
2								
3								
4								
5	$C_F\left(C_F - \frac{C_A}{2}\right) N_C$				$T_F C_F N_C$			
6								
7								
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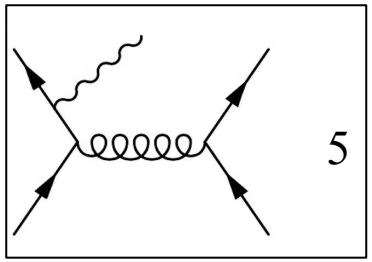
	1	2	3	4	5	6	7	8
1	Q_u^2		$Q_u Q_d$		Q_u^2 final state the same in both diagrams			
2								
3	$Q_u Q_d$		Q_d^2					
4								
5	Q_u^2 final state the same in both diagrams				Q_u^2		$Q_u Q_d$	
6								
7					$Q_u Q_d$		Q_d^2	
8								

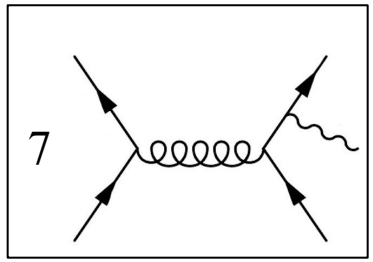


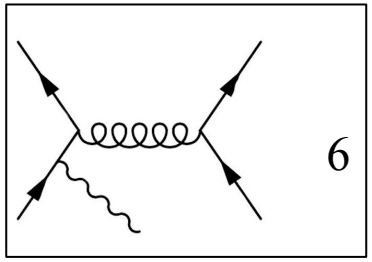


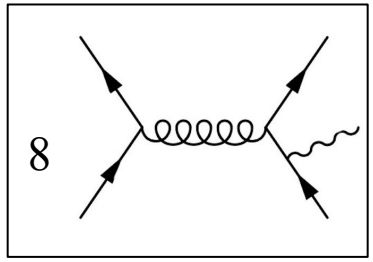










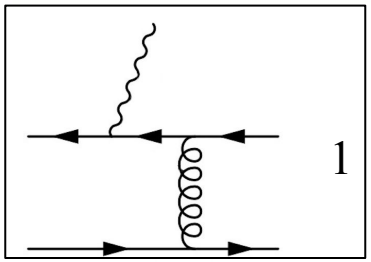


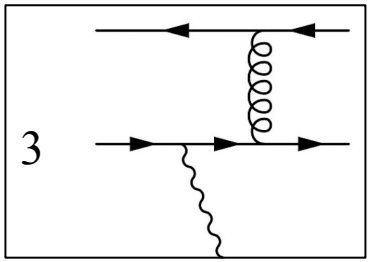
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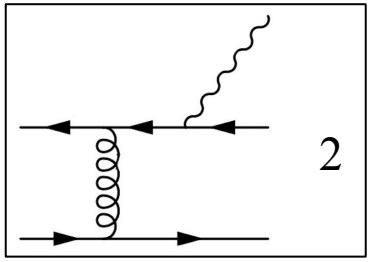
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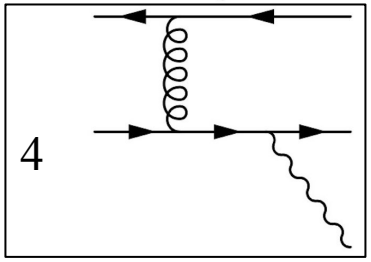
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7								
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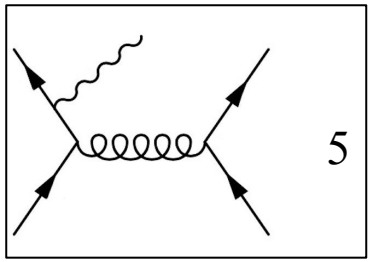
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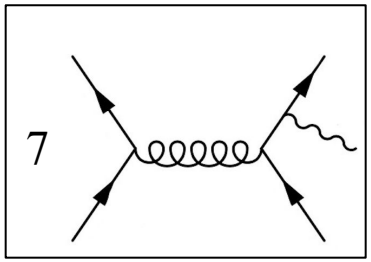


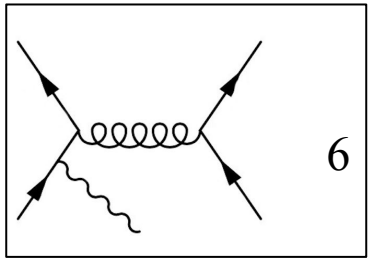


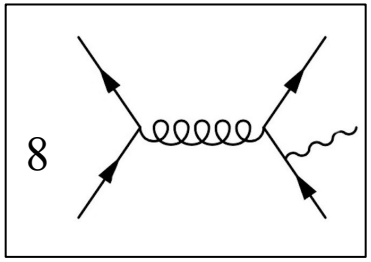












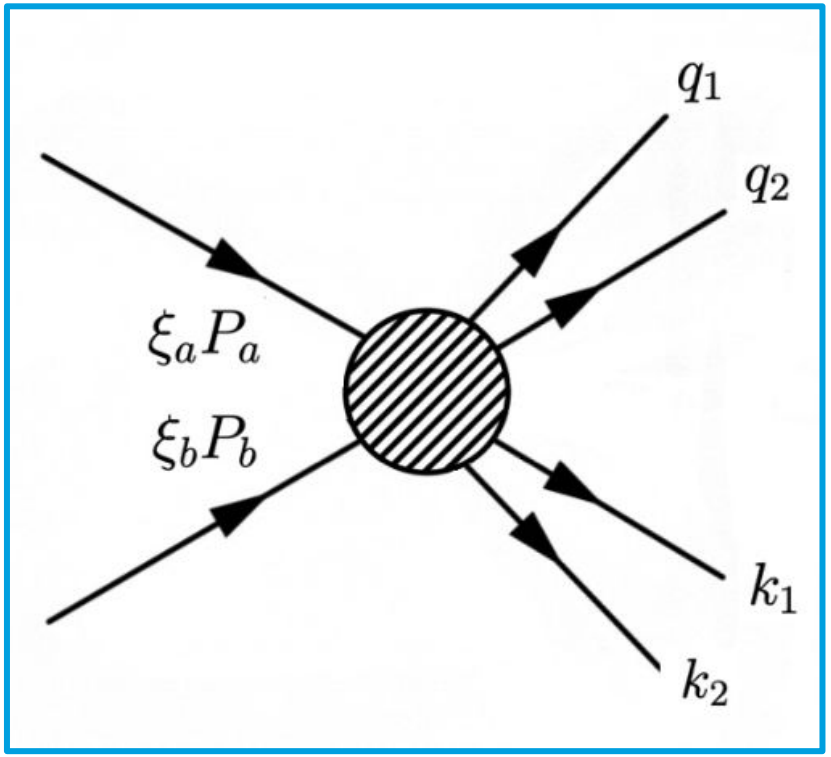
Simplifying cross section

Phase space of decay into 4 particles:

$$d\sigma_i^h \sim \int d\phi_4 W_i = \int d\phi_4 |M|_i^2 \times \text{fac.},$$

$$d\phi_4 \sim d^D k_1 \cdot d^D k_2 \cdot d^D q_1 \cdot d^D q_2$$

- ➔ Depends on all 4 external momenta;
- ➔ We need to integrate over only partonic momenta;
- ➔ Nontrivial dependence on the momenta.

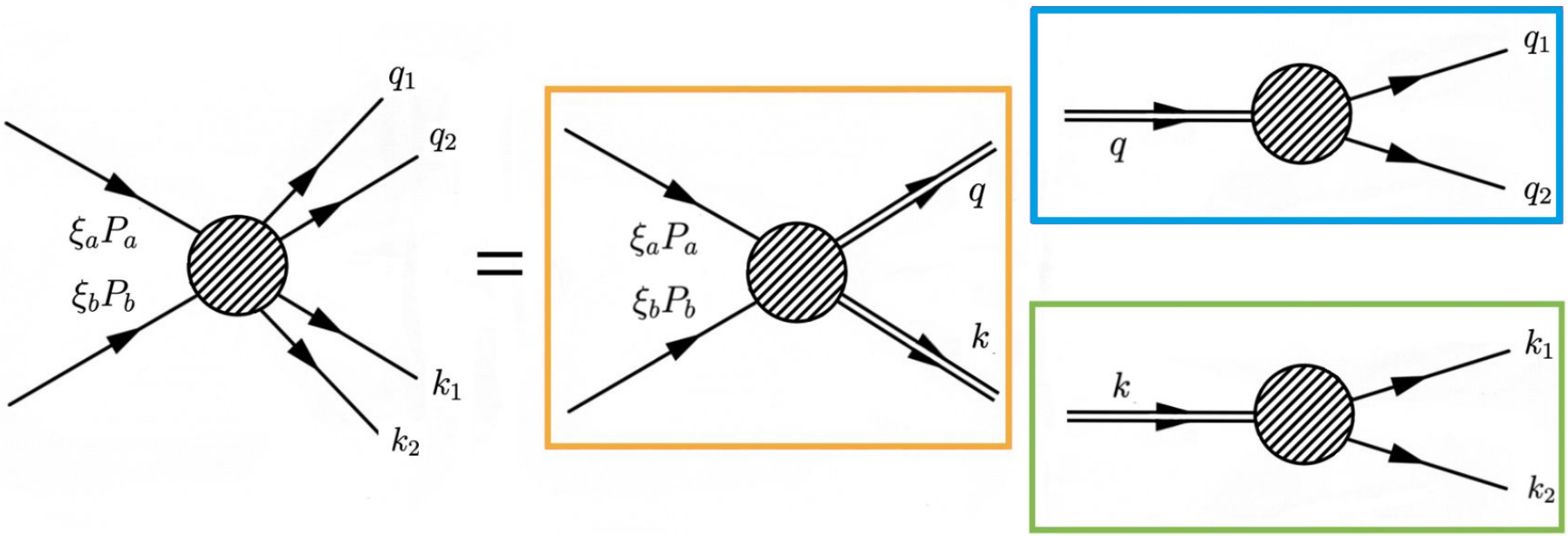


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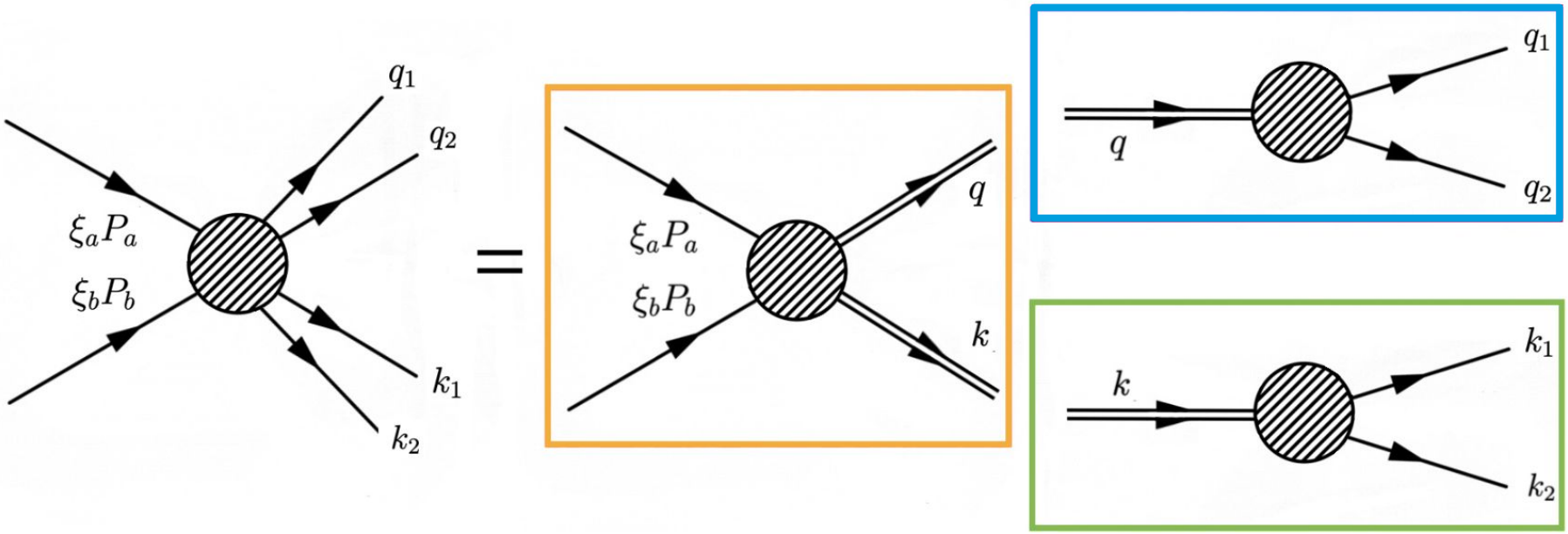
Simplifying phase space used in the cross section:



$$d\phi_4 = d\phi_{(p_a p_b \to q k)} d\phi_{(q \to q_1 q_2)} d\phi_{(k \to k_1 k_2)}$$

Simplifying cross section

Phase space of decay into 4 particles:



$$d\phi_4 = d\phi_{(p_a p_b \to qk)} d\phi_{(q \to q_1 q_2)} d\phi_{(k \to k_1 k_2)}$$

$$\sim \left(dk^- dk^+ dq^- dq^+ dq_T^2\right) \quad \sim d\Omega_q^{(3-2\epsilon)} \quad \sim d\Omega_k^{(3-2\epsilon)}$$

(Untouched)

Simplifying cross section

Final expression for the cross section:

$$d\sigma_i^h \sim \left(dk^- dk^+ dq^- dq^+ dq_T^2\right)$$ Lightcone coordinates

$$\times\, d\phi_{q\to q_1 q_2}$$ Leptonic decay phase space

$$\times \int d\phi_{k\to k_1 k_2} |M|_i^2$$ k decay phase space & matrix element

$$\times\, f_a^{[h_a]}[\xi_a, \mu_F^2] f_b^{[h_b]}[\xi_b, \mu_F^2]$$ PDF's

$$\times\, \mathrm{fac.}$$ Factor: flux, symmetry, spin, color, theta-func.

Simplifying cross section

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$$\sim d\Omega_k^{(3-2\epsilon)}$$

$$\times f_b^{[h_b]}[\xi_b, \mu_F^2]$$ PDF's

$$\times\, \mathrm{fac.}$$ Factor: flux, symmetry, spin, color, theta-func.

Integrating over angles

Master integrals:

$$\times \int d\Omega_k^{(3-2\epsilon)} \; |M|_i^2$$ k decay phase space & matrix element

Master integrals help integrate expressions like:

$$\int \frac{d\phi_{k\to k_1 k_2}}{[(p_a-k_1)^2]^{j_1}\,[(p_b-k_2)^2]^{j_2}}$$

$$\sim \int \frac{d\Omega^{(3-2\epsilon)}}{(1-\beta_1\cos\theta_2\sin\theta_1-\beta_2\cos\theta_1)^{j_1}(1-\beta_3\cos\theta_2\sin\theta_1-\beta_4\cos\theta_1)^{j_2}}$$

"Angular Integrals in d Dimensions" by G. Somogyi

arXiv:1101.3557

Integrating over angles

Partial fraction decomposition:

Can have up to 4 unique denominators: $$\int \frac{d\phi_{k\to k_1 k_2}}{2\pi} \underbrace{\frac{1}{(\text{den } 1)^{j_1}(\text{den } 2)^{j_2}(\text{den } 3)^{j_3}(\text{den } 4)^{j_4}}}$$

$$\frac{c_{12}}{(\text{den } 1)^{j_{12}}(\text{den } 2)^{j_{21}}} + \frac{c_{13}}{(\text{den } 1)^{j_{13}}(\text{den } 3)^{j_{31}}} + \frac{c_{14}}{(\text{den } 1)^{j_{14}}(\text{den } 4)^{j_{41}}} + \frac{c_{23}}{(\text{den } 2)^{j_{23}}(\text{den } 3)^{j_{32}}} + \\ + \frac{c_{24}}{(\text{den } 2)^{j_{24}}(\text{den } 4)^{j_{42}}} + \frac{c_{34}}{(\text{den } 3)^{j_{34}}(\text{den } 4)^{j_{43}}}$$

$$\frac{1}{ABCD}$$

$$\frac{1}{ABC} + \frac{1}{ABD} + \frac{1}{ACD} + \frac{1}{BCD}$$

$$\frac{1}{AB} + \frac{1}{AC} + \frac{1}{BC} \cdots$$

Integrating over angles

Master integrals:

	$(p_a-k_1)^2$	$(p_a-k_2)^2$	$(p_b-k_1)^2$	$(p_b-k_2)^2$	$(q+k_1)^2$	$(q+k_2)^2$
$(p_a-k_1)^2$	One den, massless	Separable	Two den's, both massless	Two den's, both massless	Two den's, one massive	Two den's, one massive
$(p_a-k_2)^2$	Separable	One den, massless	Two den's, both massless	Two den's, both massless	Two den's, one massive	Two den's, one massive
$(p_b-k_1)^2$	Two den's, both massless	Two den's, both massless	One den, massless	Separable	Two den's, one massive	Two den's, one massive
$(p_b-k_2)^2$	Two den's, both massless	Two den's, both massless	Separable	One den, massless	Two den's, one massive	Two den's, one massive
$(q+k_1)^2$	Two den's, one massive	Two den's, one massive	Two den's, one massive	Two den's, one massive	One den, massive	Separable
$(q+k_2)^2$	Two den's, one massive	Two den's, one massive	Two den's, one massive	Two den's, one massive	Separable	One den, massive

All possible master integrals

2 one den massless
1 one den massive
2 two den's both massless
4 two den's one massive

→ **6 unique master integrals**

Full workflow



Compute hadronic tensor $W^{\mu\nu}$

↓

Project helicity states $(x^\mu x^\nu + y^\mu y^\nu) W_{\mu\nu}$

↓

Apply partial fraction decomposition

↓

Insert appropriate master integrals
(together with $1 \to 2$ phase space)

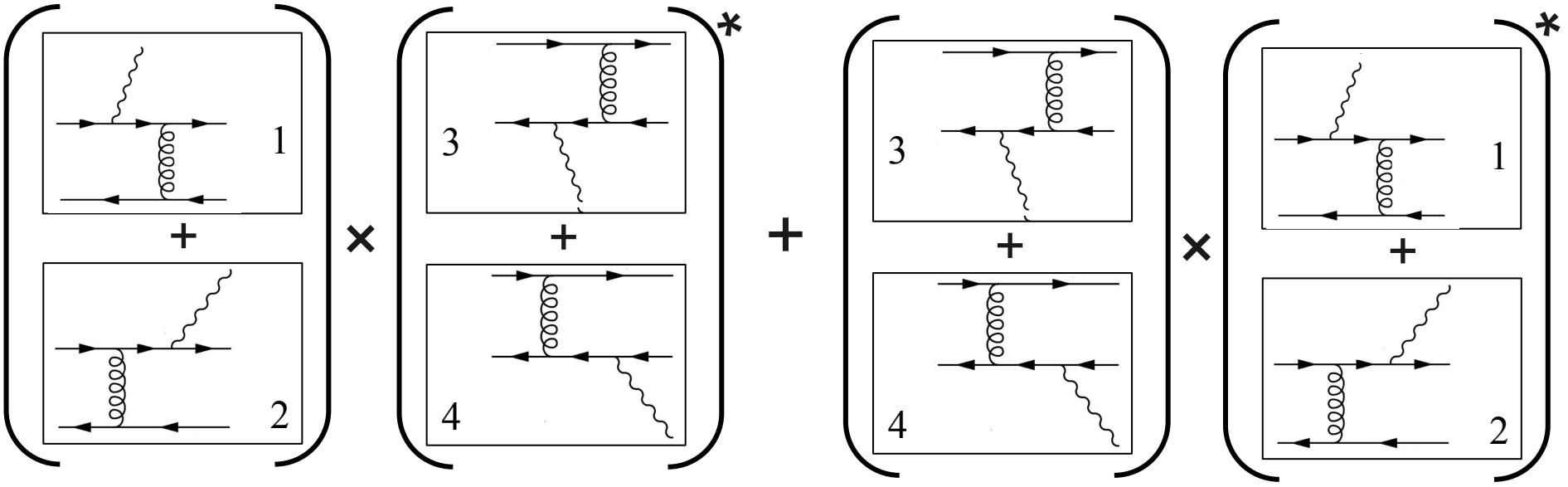
↓

Multiply with full phase space element, flux factor, spins, colors factors, PDFs and symmetry

↓

Achieve final expression for helicity cross sections $d\sigma_i^h$

Results: T-channel, color $T_F C_F N_C$, charge $Q_u Q_d$



Interference

Incoming: u, $\bar{u}$
Outgoing, u, $\bar{u}$, Z/γ*

Expect no poles for this term

Results: T-channel, color TFCFNC, charge QuQd

Two stumbling blocks:

1. $\epsilon^{\mu\nu\alpha\beta} k_{1\mu} k_{2\nu} p_{a\alpha} p_{b\beta} \equiv \epsilon^{k_1 k_2 p_a p_b}$ terms

2. γ^5 scheme in D dimensions

Results: T-channel, color TFCFNC, charge QuQd

Two stumbling blocks:

1. $\epsilon^{\mu\nu\alpha\beta} k_{1\mu} k_{2\nu} p_{a\alpha} p_{b\beta} \equiv \epsilon^{k_1 k_2 p_a p_b}$ terms

2. γ^5 scheme in D dimensions

$$\int d\phi_{(k \to k_1 k_2)} \epsilon^{k_1 k_2 p_a p_b} = 0$$

Results: T-channel, color TFCFNC, charge QuQd

Two stumbling blocks:

1. $\epsilon^{\mu\nu\alpha\beta} k_{1\mu} k_{2\nu} p_{a\alpha} p_{b\beta} \equiv \epsilon^{k_1 k_2 p_a p_b}$ terms

2. γ^5 scheme in D dimensions

$$\gamma^5 = -\frac{i}{4!}\epsilon^{abcd}\gamma_a\gamma_b\gamma_c\gamma_d$$

Problem: sometimes gives extra non-zero terms that break hermiticity of hadronic tensor (i.e. gives $W_5, W_6, W_7 \neq 0$)

Want W_5, W_6, W_7 to be 0, since they are imaginary and all others are purely real for RR.

Results: T-channel, color TFCFNC, charge QuQd

Results:

- No poles;
- Inclusive cross section coincides with the one calculated in previous studies

$$u\bar{u} \to \gamma^* X$$

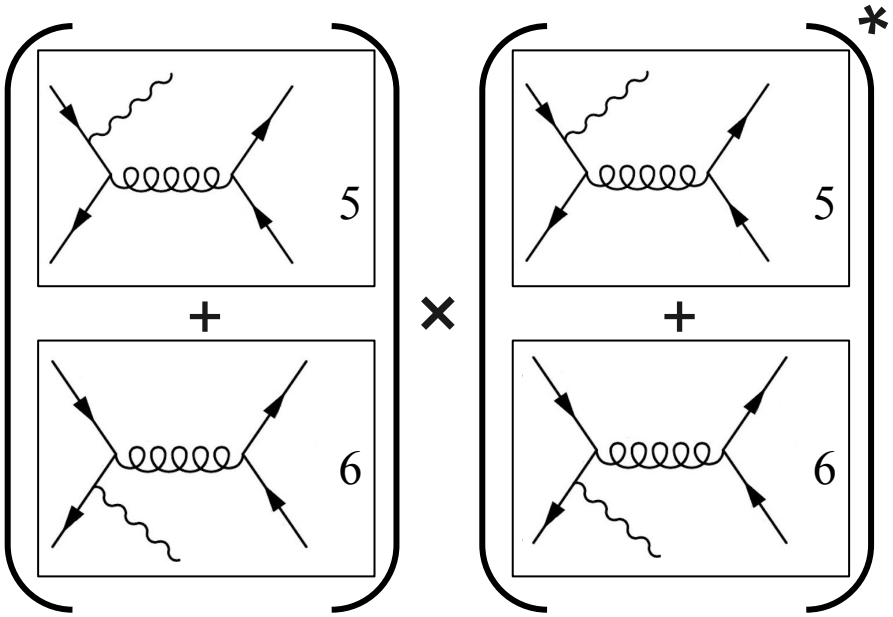
$$W^{\mu\nu} = W^{\nu\mu}$$
$$W_{-1}, W_0, W_1, W_2 \neq 0,$$
$$W_3, W_4, W_7 = 0,$$
$$W_5, W_6 = 0.$$

$$u\bar{u} \to ZX$$

$$W^{\mu\nu} \neq W^{\nu\mu}$$
$$W_{-1}, W_0, W_1, W_2 \neq 0,$$
$$W_3, W_4 \neq 0,$$
$$W_5, W_6, W_7 = 0.$$

All of the helicity cross sections saved in common repository

Results: S-channel, color $T_F C_F N_C$, charge Q_u^2



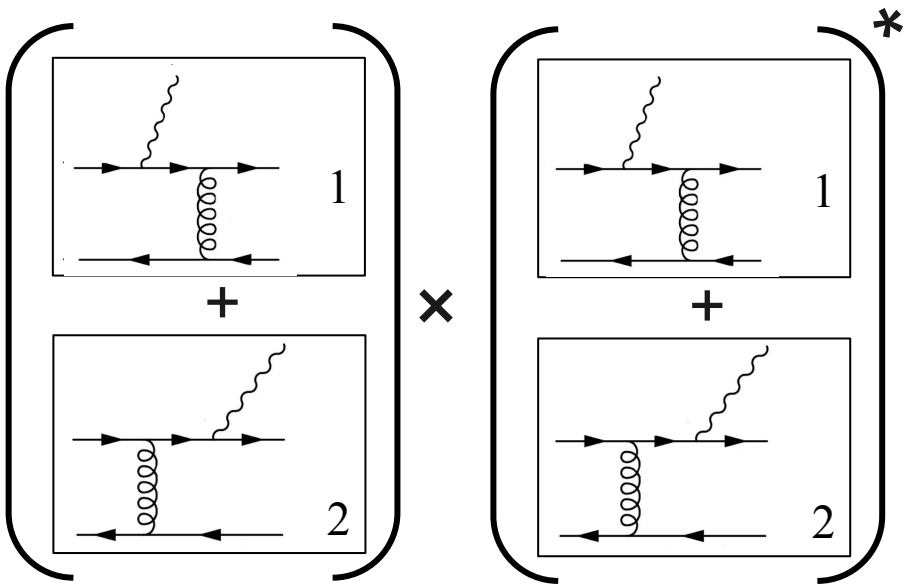
Incoming: u, $\bar{u}$
Outgoing, u, $\bar{u}$, Z/γ*

Expect poles for this term

$$k^2 = (k_1 + k_2)^2 \to 0$$

$$\frac{1}{(k^2)^{1+\epsilon}} = -\frac{\delta(k^2)}{\epsilon} + \sum_{n=0}^{\infty} (-1)^n \mathcal{L}_n(k^2) \frac{\epsilon^n}{n!}$$

Results: S-channel, color TFCFNC, charge Qu^2



Incoming: u, $\bar{u}$
Outgoing, u, $\bar{u}$, Z/γ*

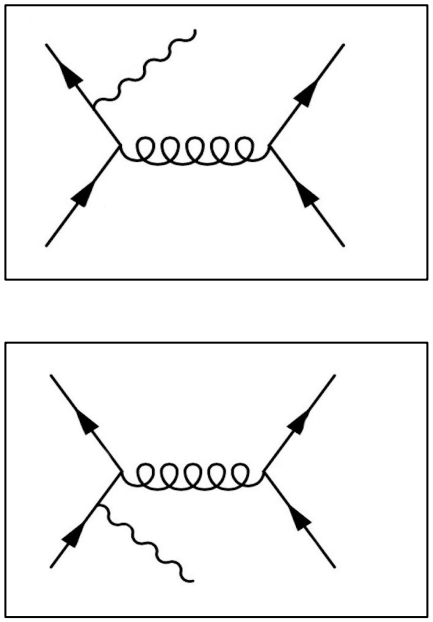
Expect poles for this term

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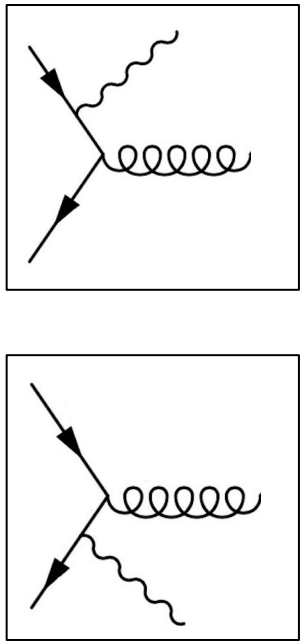
$$\frac{1}{(k^2)^{1+\epsilon}} = -\frac{\delta(k^2)}{\epsilon} + \sum_{n=0}^{\infty} (-1)^n \mathcal{L}_n(k^2) \frac{\epsilon^n}{n!}$$

Results: S-channel, color TFCFNC, charge Qu^2

Special case



$$\frac{1}{\epsilon_{IR}} +$$



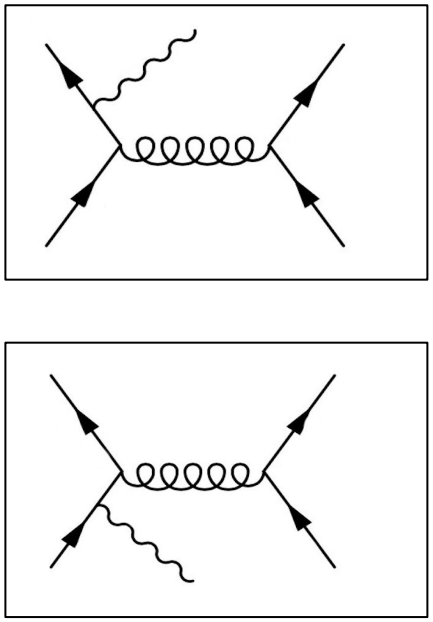
$$\frac{1}{\epsilon_{UV}}$$

$$\frac{1}{(k^2)^{1+\epsilon}} = -\frac{\delta(k^2)}{\epsilon} + \sum_{n=0}^{\infty}(-1)^n \mathcal{L}_n(k^2)\frac{\epsilon^n}{n!}$$

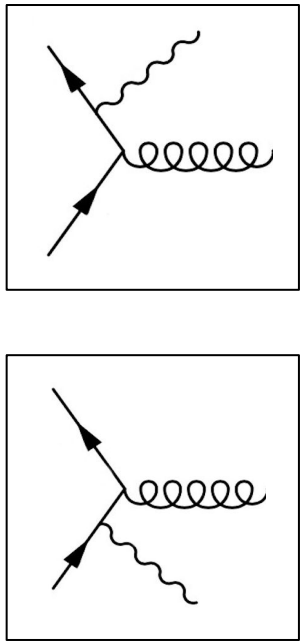
$$\frac{\alpha_0}{2\pi} \sim \frac{\alpha_s(\mu_R^2)}{2\pi}\left[1 - \frac{\beta_0}{\epsilon}\frac{\alpha_s(\mu_R^2)}{2\pi} + O(\alpha_s^2)\right]$$

Results: S-channel, color TFCFNC, charge Qu^2

Special case



$$\frac{1}{\epsilon_{IR}} +$$



$$\frac{1}{\epsilon_{UV}} = 0$$

$$\frac{1}{(k^2)^{1+\epsilon}} = -\frac{\delta(k^2)}{\epsilon} + \sum_{n=0}^{\infty}(-1)^n \mathcal{L}_n(k^2)\frac{\epsilon^n}{n!}$$

$$\frac{\alpha_0}{2\pi} \sim \frac{\alpha_s(\mu_R^2)}{2\pi}\left[1 - \frac{\beta_0}{\epsilon}\frac{\alpha_s(\mu_R^2)}{2\pi} + O(\alpha_s^2)\right]$$

Conclusions and outlook

What we did:

- Calculated some of the Drell-Yan helicity cross sections up to $\mathcal{O}(\alpha_s^2)$;
- Checked the cancellation of IR poles;
- Partially compared our results with ones in literature.

What we want to do:

- Improve γ^5 scheme;
- Compare other results with literature;
- Check diagrams that would cancel with PDF renormalization;
- Move on to other color structures.

Use our analytical expressions, integrate over free variables and plot the spectrum w.r.t. q_T^2

Thank you for your attention!

Any questions?

That's me! :)



All DESY Hamburg summer students

General theory

Structure functions and projectors:

$$W^{\mu\nu} = \sum_{i=-1}^{7} P_i^{\mu\nu} W_i,$$

Helicity density matrix:

$$W_{\lambda\lambda'}(q, P_a, P_b) = \epsilon_\lambda^\mu \epsilon_{\lambda'}^{*\nu} W_{\mu\nu}(q, P_a, P_b)$$

Polarization vectors in vector boson rest frame

Structure functions:

$$W_{-1} = W_{++} + W_{--}$$
$$W_0 = 2W_{00}$$
$$W_1 = -\frac{1}{\sqrt{2}}(W_{+0} + W_{0+} + W_{-0} + W_{0-})$$
$$W_2 = -2(W_{+-} + W_{-+})$$
$$W_3 = -\sqrt{2}(W_{+0} + W_{0+} - W_{-0} - W_{0-})$$
$$W_4 = 2(W_{++} - W_{--})$$
$$W_5 = -i(W_{+-} - W_{-+})$$
$$W_6 = -\frac{i}{\sqrt{2}}(W_{+0} - W_{0+} - W_{-0} + W_{0-})$$
$$W_7 = -\sqrt{2}i(W_{+0} - W_{0+} + W_{-0} - W_{0-})$$

General theory

Structure functions and projectors:

$$W^{\mu\nu} = \sum_{i=-1}^{7} P_i^{\mu\nu} W_i,$$

Structure functions as Lorentz-scalar projections w.r.t. four orthogonal vectors:

$$t^\mu = \frac{q^\mu}{\sqrt{q^2}},$$

$$z^\mu = \frac{s_{bq} P_a^\mu - s_{aq} P_b^\mu}{\sqrt{s_{ab} s_{aq} s_{bq}}},$$

$$x^\mu = \frac{s_{aq} s_{bq} q^\mu - s_{bq} q^2 P_a^\mu - s_{aq} q^2 P_b^\mu}{\sqrt{s_{aq} s_{bq} q^2 (s_{aq} s_{bq} - s_{ab} q^2)}},$$

$$y^\mu = \epsilon^{\mu\nu\rho\sigma} t_\nu x_\rho z_\sigma,$$

$$s_{ab} \equiv 2 P_a \cdot P_b,$$
$$s_{aq} \equiv 2 P_a \cdot q,$$
$$s_{bq} \equiv 2 P_b \cdot q.$$

Depend on **three external momenta**: P_a^μ, P_b^μ, q^μ

Integrating over angles

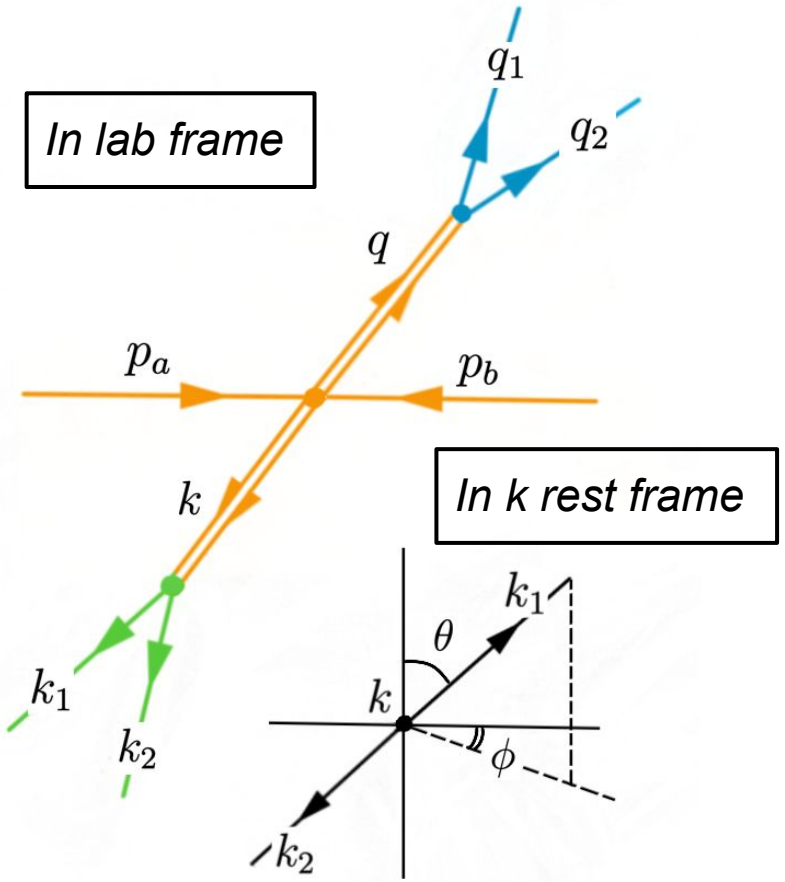
Master integrals:

$$\times \int d\phi_{k\to k_1 k_2} W_i$$

k decay phase space & matrix element

$$\sim d\Omega_k^{(3-2\epsilon)}$$

D-1 angles w.r.t k:



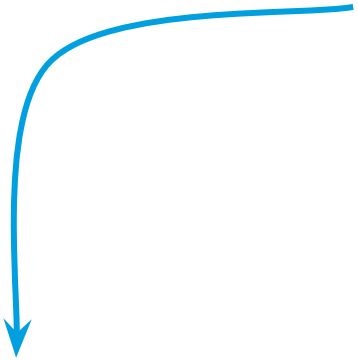
General theory

Structure functions and projectors:

$$W^{\mu\nu} = \sum_{i=-1}^{7} P_i^{\mu\nu} W_i,$$

Structure functions as Lorentz-scalar projections w.r.t. four orthogonal vectors:

$$W_{-1} = (x^\mu x^\nu + y^\mu y^\nu) W_{\mu\nu},$$
$$W_0 = 2 z^\mu z^\nu W_{\mu\nu},$$
$$W_1 = -(x^\mu z^\nu + z^\mu x^\nu) W_{\mu\nu},$$
$$W_2 = -2(x^\mu x^\nu - y^\mu y^\nu) W_{\mu\nu},$$
$$W_3 = 2i(y^\mu z^\nu - z^\mu y^\nu) W_{\mu\nu},$$
$$W_4 = 2i(x^\mu y^\nu - y^\mu x^\nu) W_{\mu\nu},$$
$$W_5 = -(x^\mu y^\nu + y^\mu x^\nu) W_{\mu\nu},$$
$$W_6 = -(y^\mu z^\nu + z^\mu y^\nu) W_{\mu\nu},$$
$$W_7 = -2i(x^\mu z^\nu - z^\mu x^\nu) W_{\mu\nu}.$$



Angular coefficients

Integrating over angles

Master integrals:

$$\int \frac{d\phi_{k\to k_1 k_2}}{2\pi} \frac{1}{[(p_a-k_1)^2]^{j_1}\,[(p_b-k_2)^2]^{j_2}} = \int \frac{d\phi_{k\to k_1 k_2}}{2\pi} \frac{1}{[(p_a-k_2)^2]^{j_1}\,[(p_b-k_1)^2]^{j_2}} =$$

$$(-1)^{j_1+j_2}\mu^{2\epsilon}\frac{(4\pi)^\epsilon}{(k^+k^- - q_T^2)^\epsilon}\left[k^+(k^-+q^-)\right]^{-j_1}\left[k^-(k^++q^+)\right]^{-j_2}$$

$$\times \frac{1}{(4\pi)^2}\frac{\Gamma(1-j_1-\epsilon)\Gamma(1-j_2-\epsilon)}{\Gamma(1-\epsilon)\Gamma(2-j_1-j_2-2\epsilon)}\,{}_2F_1\left(j_1,j_2;1-\epsilon;1-\frac{q_T^2}{k^-k^+}\right)$$

Example of a master integral for two ‘massless’ denominators

“Angular Integrals in d Dimensions” by G. Somogyi

arXiv:1101.3557

Integrating over angles

Partial fraction decomposition:

Can have up to 4 unique denominators: $$\int \frac{d\phi_{k\to k_1 k_2}}{2\pi} \frac{1}{(\text{den } 1)^{j_1}(\text{den } 2)^{j_2}(\text{den } 3)^{j_3}(\text{den } 4)^{j_4}}$$

$$\frac{c_{12}}{(\text{den } 1)^{j_{12}}(\text{den } 2)^{j_{21}}} + \frac{c_{13}}{(\text{den } 1)^{j_{13}}(\text{den } 3)^{j_{31}}} + \frac{c_{14}}{(\text{den } 1)^{j_{14}}(\text{den } 4)^{j_{41}}} + \frac{c_{23}}{(\text{den } 2)^{j_{23}}(\text{den } 3)^{j_{32}}} + \\ + \frac{c_{24}}{(\text{den } 2)^{j_{24}}(\text{den } 4)^{j_{42}}} + \frac{c_{34}}{(\text{den } 3)^{j_{34}}(\text{den } 4)^{j_{43}}}$$

Use partial fraction decomposition: $$\frac{1}{\prod_{k=1}^{n}\left(\sum_{i=1}^{m} M_{ki}x_i\right)} = \sum_{j=1}^{n} \frac{c_j}{\prod_{\substack{k=1\\k\neq j}}^{n}\left(\sum_{i=1}^{m} M_{ki}x_i\right)}$$

$$\Rightarrow \sum_{j=1}^{n}\sum_{i=1}^{m} c_j M_{ji} x_i = 1. \Rightarrow \sum_{j=1}^{n} c_j M_{ji} = \delta_{mi}$$

Integrating over angles

Partial fraction decomposition:

Use partial fraction decomposition: $$\frac{1}{\prod_{k=1}^{n}\left(\sum_{i=1}^{m} M_{ki}x_i\right)} = \sum_{j=1}^{n} \frac{c_j}{\prod_{\substack{k=1\\k\neq j}}^{n}\left(\sum_{i=1}^{m} M_{ki}x_i\right)} \quad \{x_i\}, x_1 = 1$$

$$\Rightarrow \sum_{j=1}^{n}\sum_{i=1}^{m} c_j M_{ji} x_i = 1. \Rightarrow \sum_{j=1}^{n} c_j M_{ji} = \delta_{mi}$$

In our case x's are independent functions of angles θ ϕ and denominators can be written as:

$$(p_a - k_1)^2 = -\frac{(k^- + q^-)}{2} k^+ (1 - x),$$
$$(p_a - k_2)^2 = -\frac{(k^- + q^-)}{2} k^+ (1 + x),$$
$$(p_b - k_1)^2 = -\frac{(k^+ + q^+)}{2} k^- (1 - y),$$
$$\dots$$

Integrating over angles

Partial fraction decomposition:

Use partial fraction decomposition:

$$\frac{1}{\prod_{k=1}^{n}\left(\sum_{i=1}^{m} M_{ki}x_i\right)} = \sum_{j=1}^{n} \frac{c_j}{\prod_{\substack{k=1\\k\neq j}}^{n}\left(\sum_{i=1}^{m} M_{ki}x_i\right)}$$

$$\Rightarrow \sum_{j=1}^{n}\sum_{i=1}^{m} c_j M_{ji} x_i = 1. \Rightarrow \sum_{j=1}^{n} c_j M_{ji} = \delta_{mi}$$

$$\frac{1}{ABCD}$$

$$\frac{1}{ABC} + \frac{1}{ABD} + \frac{1}{ACD} + \frac{1}{BCD}$$

$$\frac{1}{AB} + \frac{1}{AC} + \frac{1}{BC} \cdots$$

Results: T-channel, color TFCFNC, charge QuQd

Two stumbling blocks:

1. $\epsilon^{\mu\nu\alpha\beta} k_{1\mu} k_{2\nu} p_{a\alpha} p_{b\beta} \equiv \epsilon^{k_1 k_2 p_a p_b}$ terms

2. γ^5 scheme in D dimensions

$$\int d\phi_{(k \to k_1 k_2)} \epsilon^{k_1 k_2 p_a p_b} = 0$$

Results: T-channel, color TFCFNC, charge QuQd

Results:

- No poles;
- Inclusive cross section coincides with the one calculated in previous studies

$$u\bar{u} \to \gamma^* X$$

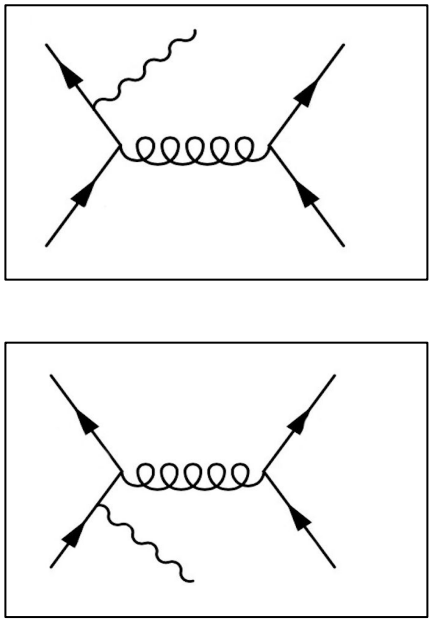
$$W^{\mu\nu} = W^{\nu\mu}$$
$$W_{-1}, W_0, W_1, W_2 \neq 0,$$
$$W_3, W_4, W_7 = 0,$$
$$W_5, W_6 = 0.$$

$$u\bar{u} \to ZX$$

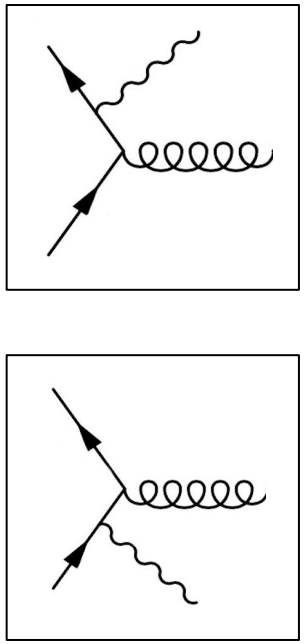
$$W^{\mu\nu} = v_f^2 W_1^{\mu\nu} + v_f a_f W_2^{\mu\nu} + a_f^2 W_3^{\mu\nu},$$
$$W_{-1}, W_0, W_1, W_2 \sim (v_f^2 + a_f^2),$$
$$W_3, W_4 \sim v_f a_f.$$

Results: S-channel, color $T_F C_F N_C$, charge Q_{2u}

Special case



$$\frac{1}{\epsilon_{IR}} +$$



$$\frac{1}{\epsilon_{UV}}$$

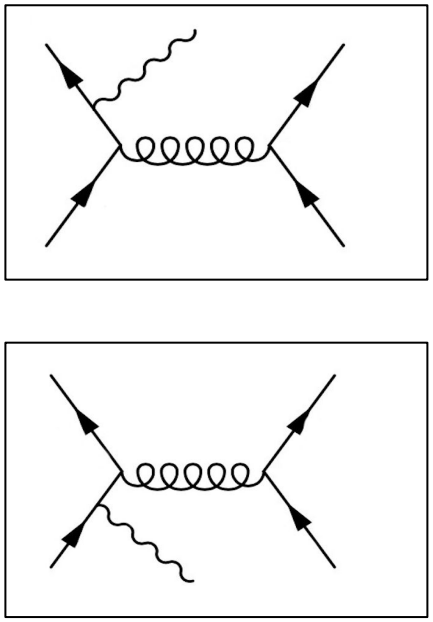
$$\frac{1}{(k^2)^{1+\epsilon}} = -\frac{\delta(k^2)}{\epsilon} + \sum_{n=0}^{\infty}(-1)^n \mathcal{L}_n(k^2)\frac{\epsilon^n}{n!}$$

$$\frac{\alpha_0}{2\pi} \sim \frac{\alpha_s(\mu_R^2)}{2\pi}\left[1 - \frac{\beta_0}{\epsilon}\frac{\alpha_s(\mu_R^2)}{2\pi} + O(\alpha_s^2)\right]$$

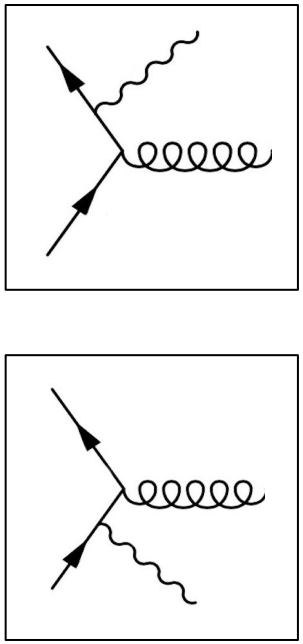
$$\beta_0 = \frac{11}{6}C_A - \frac{2}{3}T_F n_f$$

Results: S-channel, color $T_F C_F N_C$, charge Q_{2u}

Special case



$$\frac{1}{\epsilon_{IR}} +$$



$$\frac{1}{\epsilon_{UV}} = 0$$

$$\frac{1}{(k^2)^{1+\epsilon}} = -\frac{\delta(k^2)}{\epsilon} + \sum_{n=0}^{\infty}(-1)^n \mathcal{L}_n(k^2)\frac{\epsilon^n}{n!}$$

$$\sim n_f$$

$$\frac{\alpha_0}{2\pi} \sim \frac{\alpha_s(\mu_R^2)}{2\pi}\left[1 - \frac{\beta_0}{\epsilon}\frac{\alpha_s(\mu_R^2)}{2\pi} + O(\alpha_s^2)\right]$$

$$\beta_0 = \frac{11}{6}C_A - \frac{2}{3}T_F n_f$$

Results: S-channel, color TFCFNC, charge Q2u

Results:

- Poles cancel with single real.

$$u\bar{u} \to \gamma^* X$$

$$W^{\mu\nu} = W^{\nu\mu}$$
$$W_{-1}, W_0, W_1, W_2 \neq 0,$$
$$W_3, W_4, W_7 = 0,$$
$$W_5, W_6 = 0.$$

$$u\bar{u} \to ZX$$

$$W^{\mu\nu} \neq W^{\nu\mu}$$
$$W_{-1}, W_0, W_1, W_2 \neq 0,$$
$$W_3, W_4 \neq 0,$$
$$W_5, W_6, W_7 = 0.$$

$$\int d\phi_{(k\to k_1 k_2)} \epsilon^{k_1 k_2 p_a p_b} = 0$$

Proof:

p_a^μ, p_b^μ in k rest frame have at most two non-zero spatial components:

$$f(\theta_1, \theta_2)$$

$\epsilon^{\mu\nu\rho\sigma} p_{a\nu} p_{b\rho} k_{1\sigma}$ is orthogonal to p_a^μ, p_b^μ so it has a third non-zero component:

$$\epsilon^{k_1 k_2 p_a p_b} = \tilde{f}(\theta_3, \theta_2, \theta_1)$$

$$\int \int f(\theta_1, \theta_2) \int_0^\pi d\theta_3 \cos\theta_3 \sin^{-1-2\epsilon}\theta_3 = 0.$$