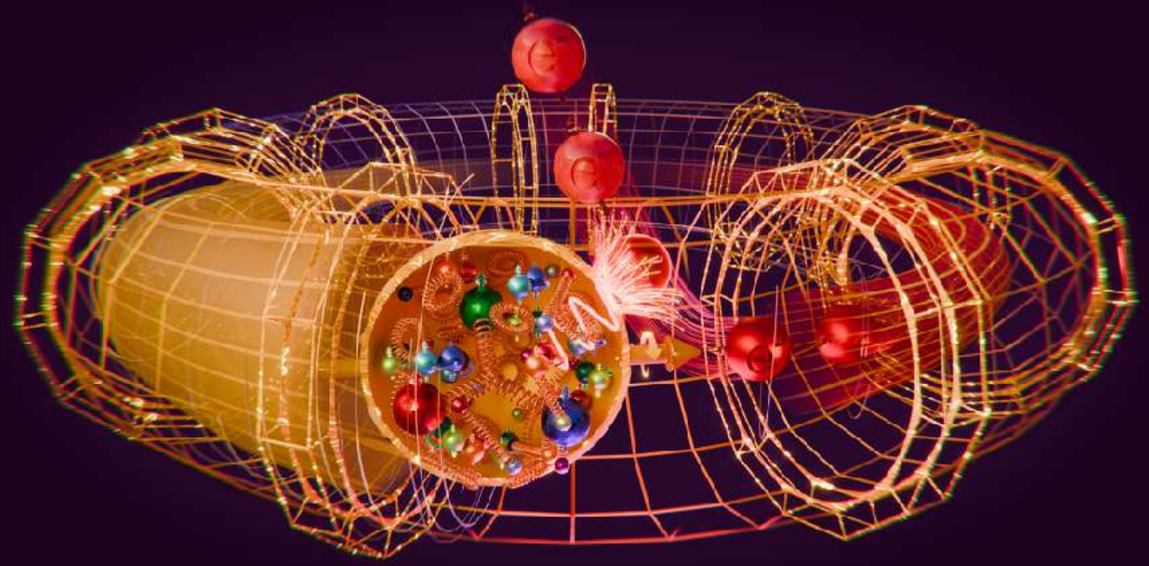
Forward Neutrons in Electron–Proton Colliders



Daria Trotsenko
Harvard University

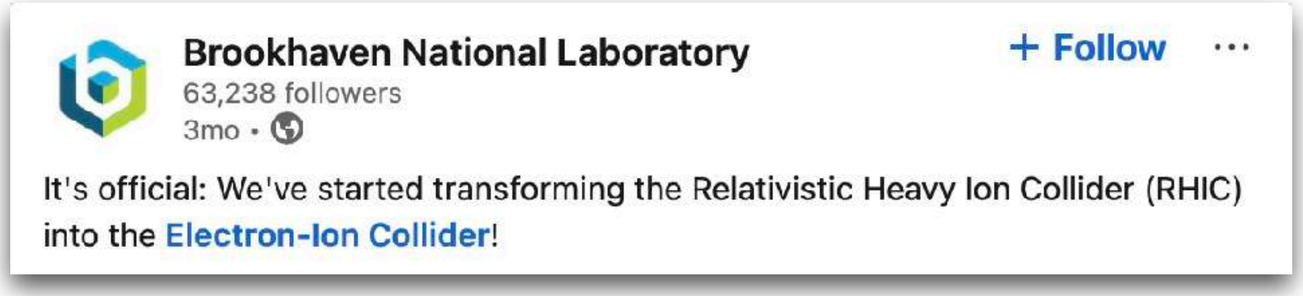
HARVARD UNIVERSITY

August 12th 2026
T-2 & P-3 Seminar, LANL

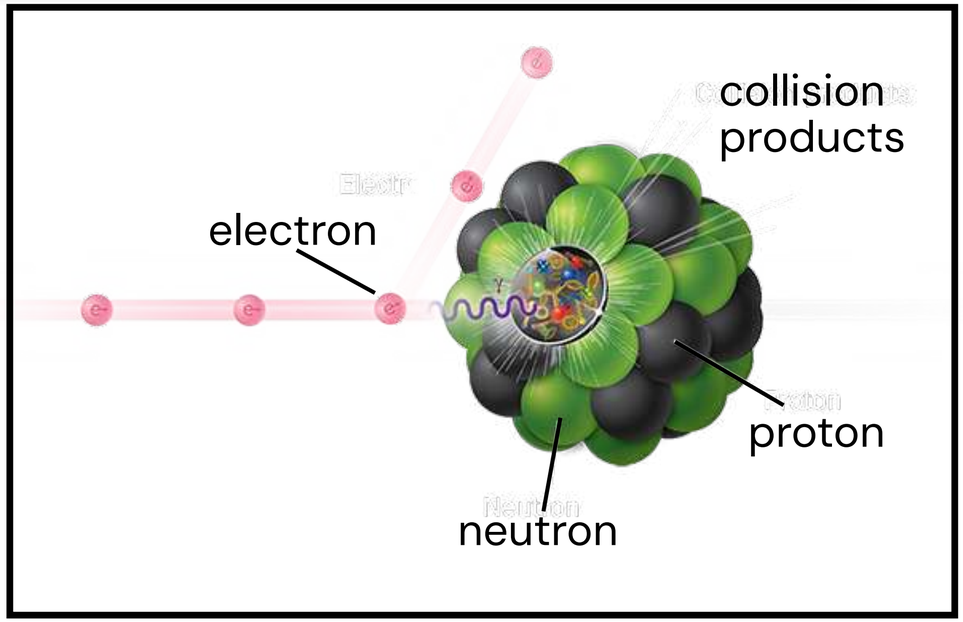
Los Alamos NATIONAL LABORATORY

Electron-Ion Collider at Brookhaven

EIC has **CD-3B approval** (long lead procurement and site preparation)!







$1.7-2.88 billion dollar project

Polarized $e+p,\ e+A$

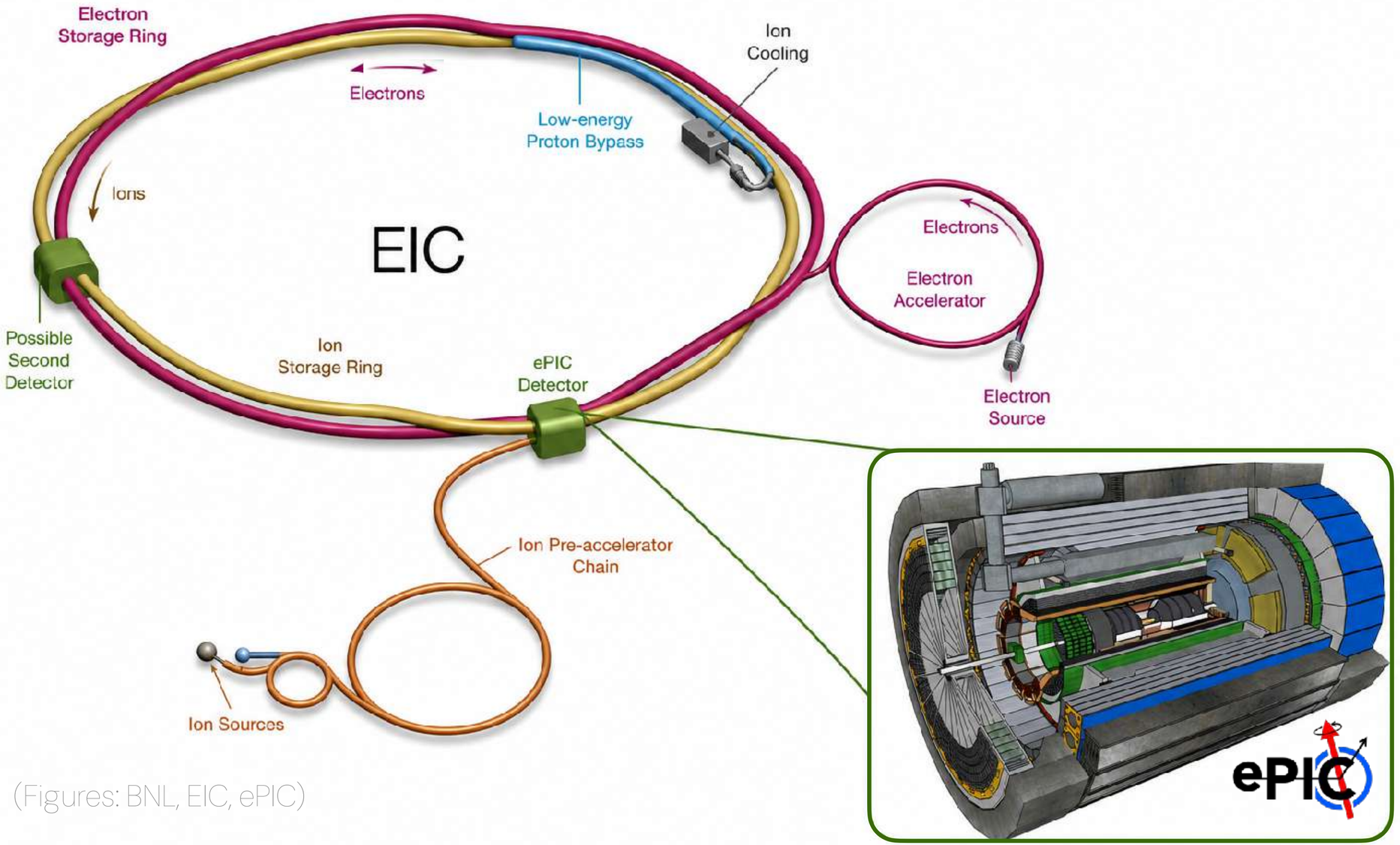
COM energy 20-140 GeV

First light: 2030s

Access to 3D partonic structure and small-x gluon dynamics

(Figures: BNL, EIC)

Electron–Proton/Ion Collider

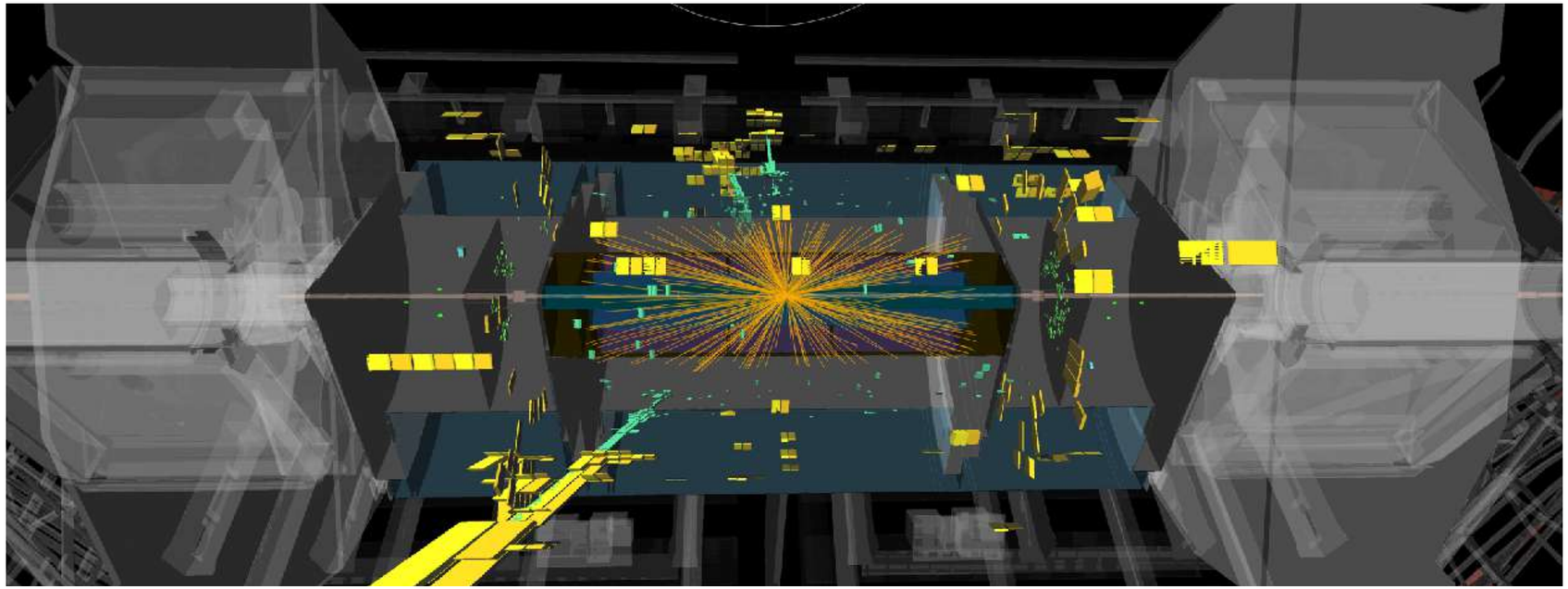


(Figures: BNL, EIC, ePIC)

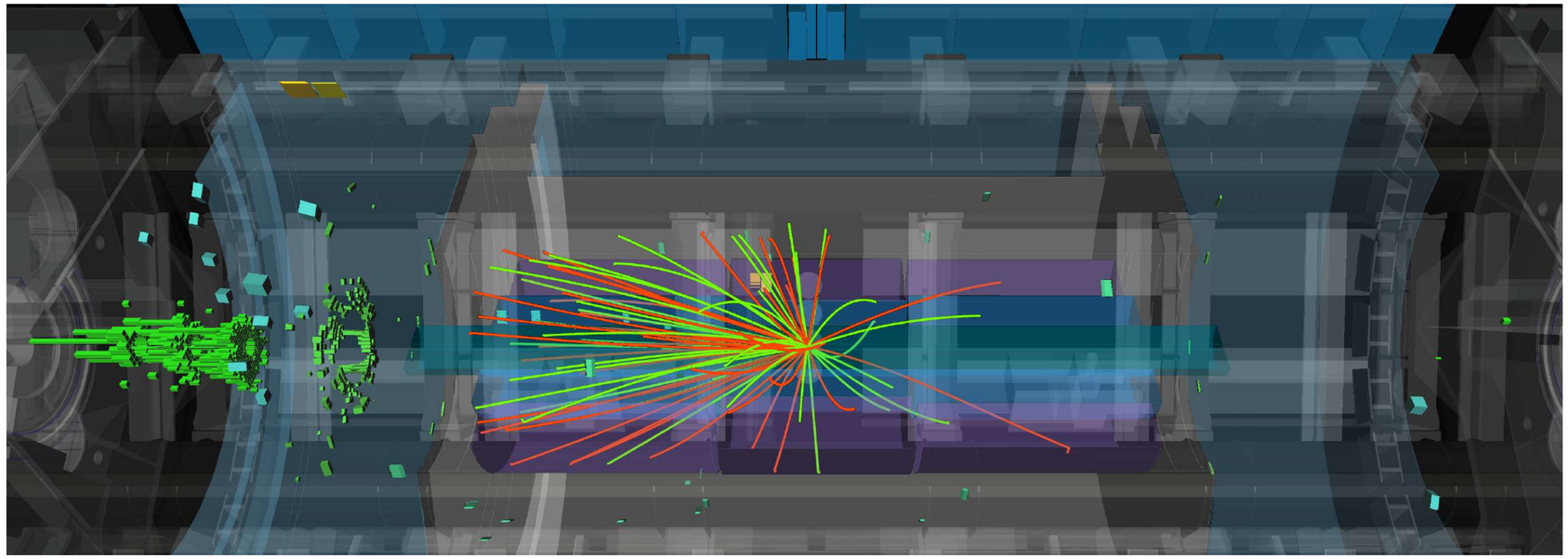
- First EIC experiment
- Compact (~10 m)
- Large acceptance detector (3°–177°)
- Hermetic recoil tagging and exclusive final-state reconstruction

Central detector alone is not enough — far-forward system has also interesting physics!

Diffractive processes



Normal collision



Diffractive collision

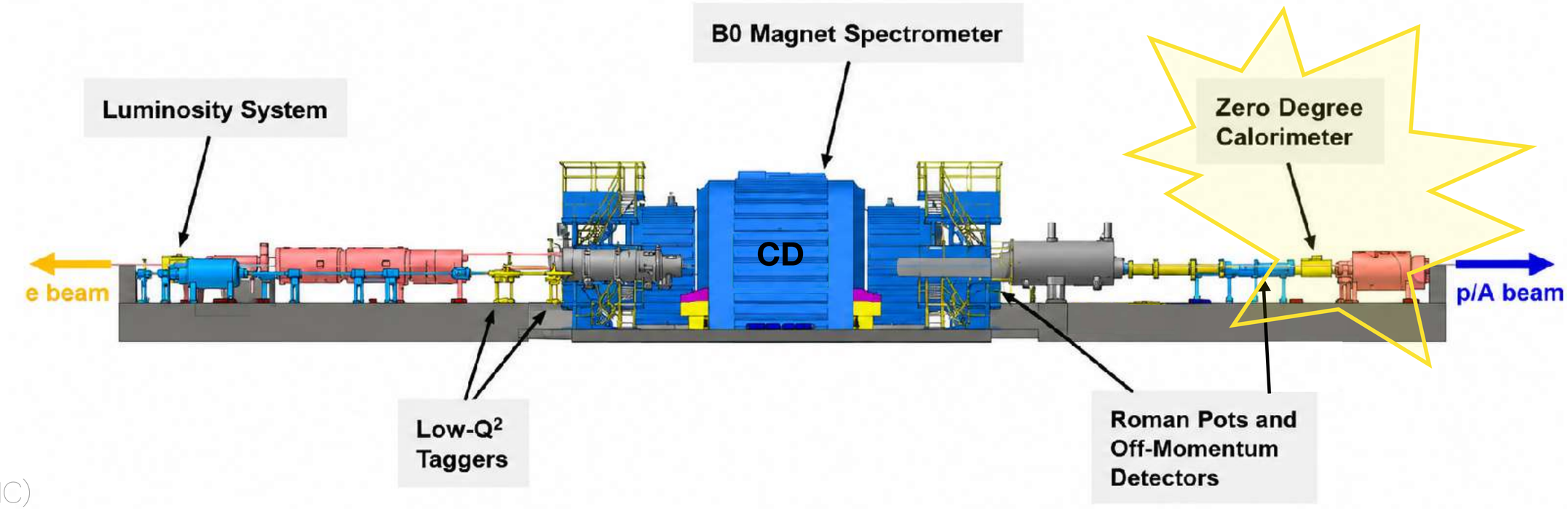
forward + rapidity gap

(Image: CERN)

Electron–Proton/Ion Collider

Zero Degree Calorimeter:

- 35.8 m from the int. point
- measures n, γ high rapidities $\eta > 6$
- measure forward particles with energies up to 275 GeV
- transverse-momentum resolution of ~8.4 MeV



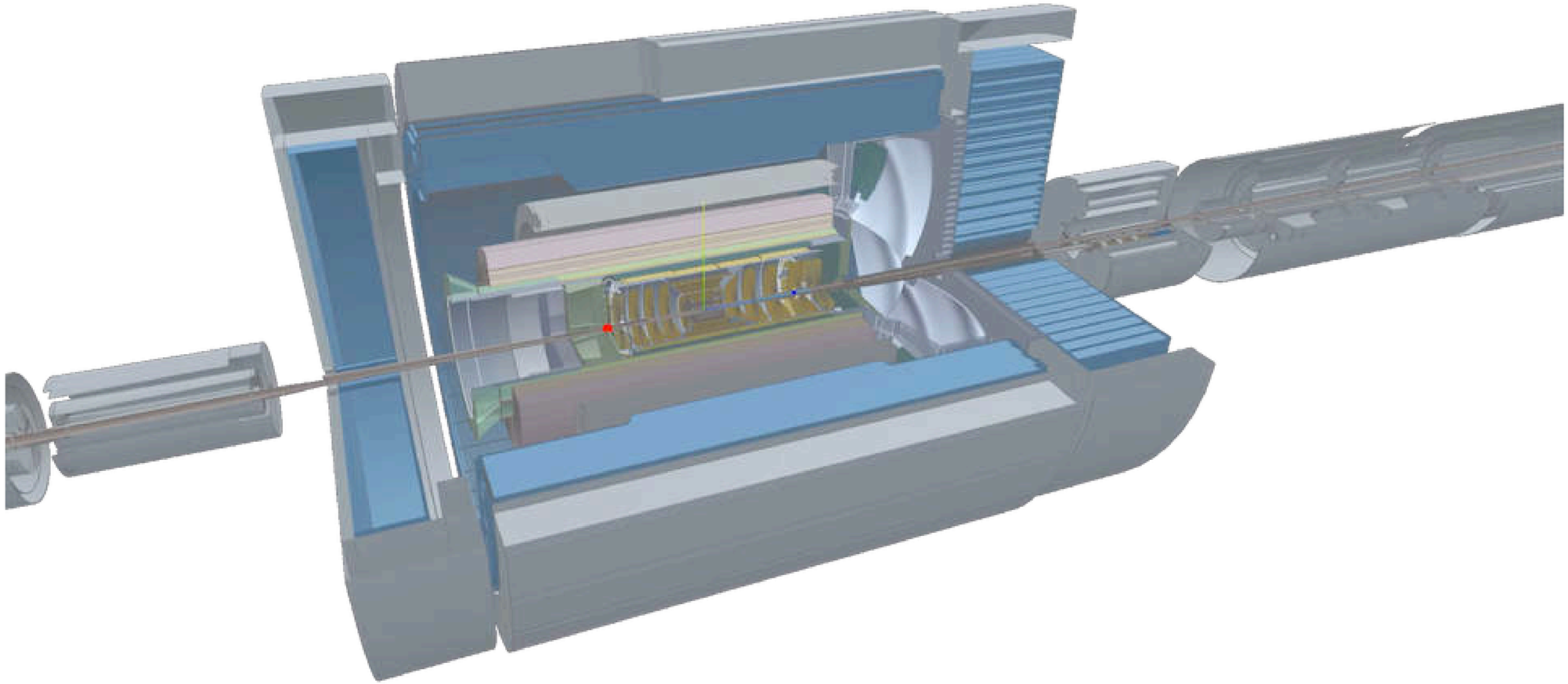
(Figure: ePIC)

time is right to start developing the theory to match the future data

Forward neutrons at the EIC

$$ep \to eXn$$

*Very unusual topology: **quark change**, big **rapidity gap** with very **forward** isolated neutrons*



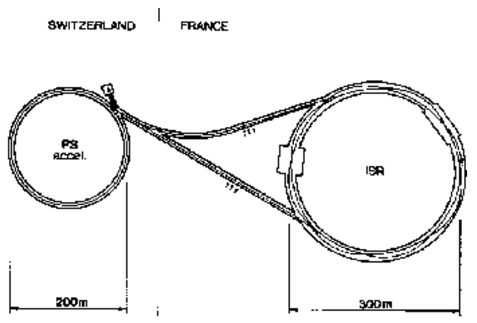
(Dmitry Romanov/Thomas Jefferson National Accelerator Facility)

Forward neutrons at other experiments

$$pp \to XYn$$

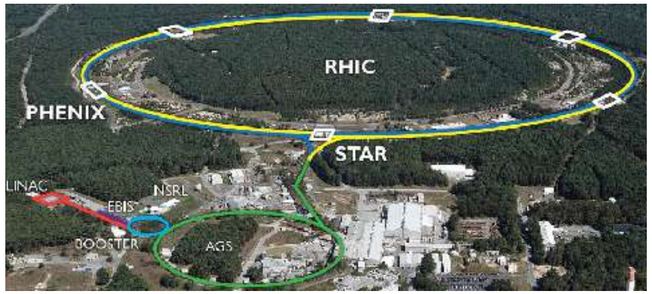
1976

ISR



2006

RHIC (PHENIX)



2010 2015

LHC (LHCf 7 TeV) LHC (LHCf 13 TeV)



2017

RHIC (RHICf)

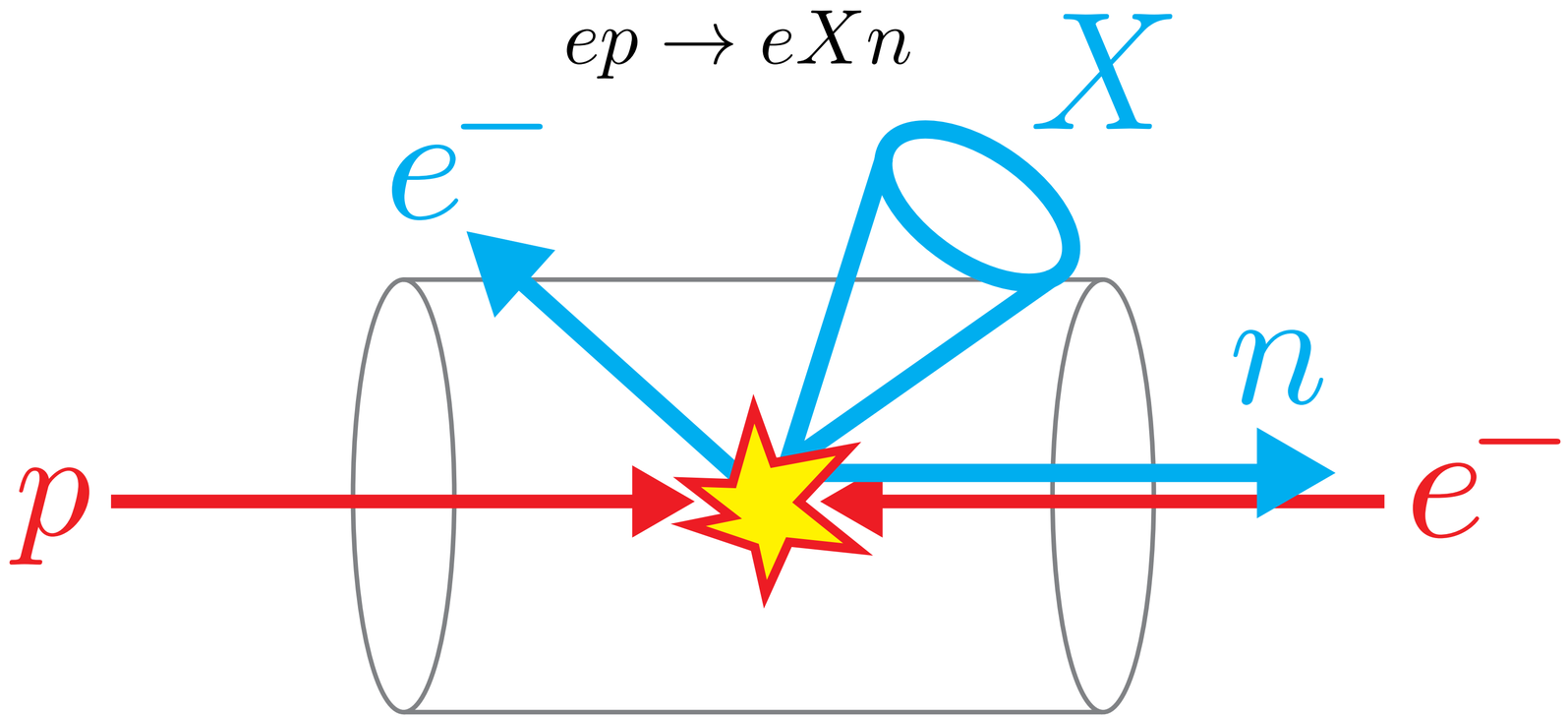
1992–1997 2006–2007

HERA (H1 & ZEUS) HERA-II



$$ep \to eXn$$

Forward neutrons at the EIC



- ~2–8% of the ep events at HERA had forward neutrons

  230 000 out of 9.3 million DIS events!

- ~8–20%* of the ep events estimated to reach ZDC in EIC

  *this number is based on extrapolating the HERA data **can be also calculated from CDM

(H1 Collaboration, 1001.0532, 1404.0201, ZEUS hep-ex/0702028)
(*Y. Yamazaki, "Neutron angular distribution and dose in ZDC") (**Carvalho1, Gonçalves, Navarra, Spiering 2011.09515)

Why study forward neutrons at the EIC?

Sullivan process: $ep \to eXn$ *is thought to be mediated by a* ***pion***

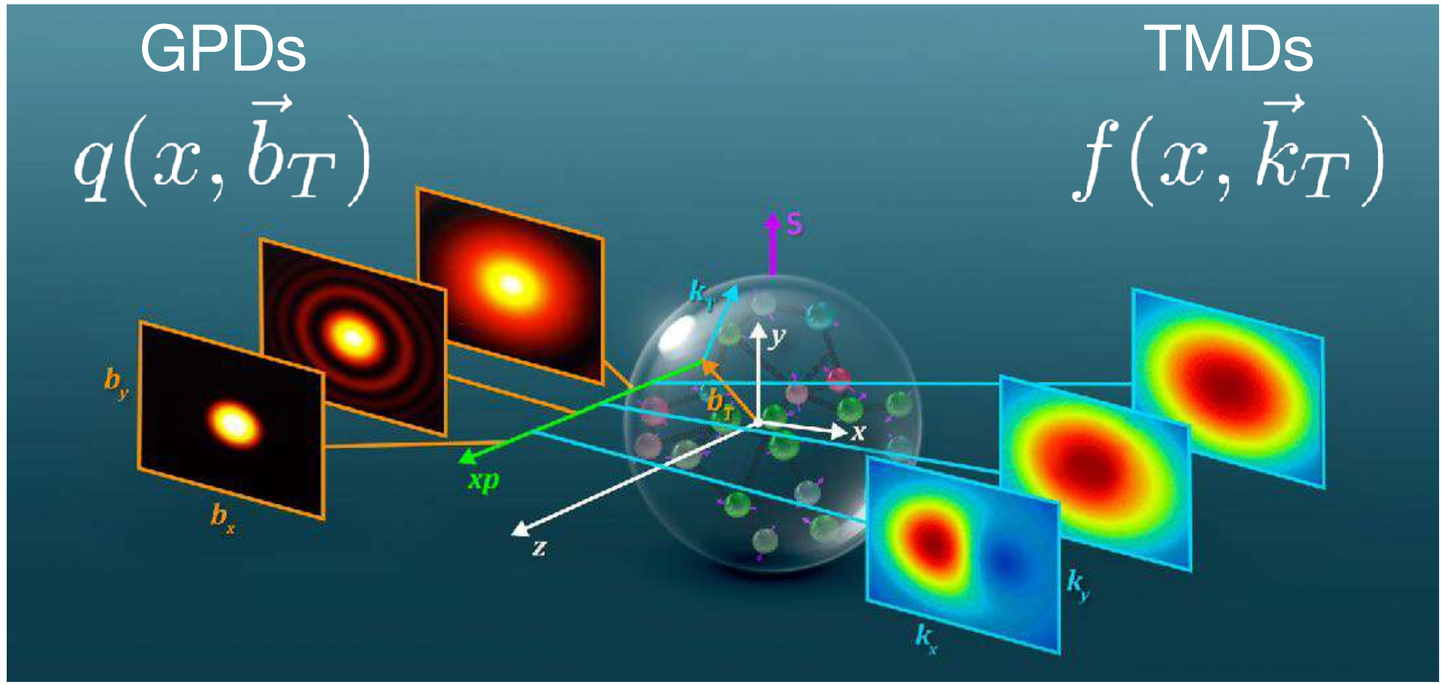
$$e^- p \to e^- n\pi^+ \xrightarrow{\gamma^* \pi^+ \to X} e^- X n$$

- Can access pion's **Generalized Parton Distributions**?
- Learn about **pion's emergent mass**:

Higgs vs QCD contributions?

(lattice picture)

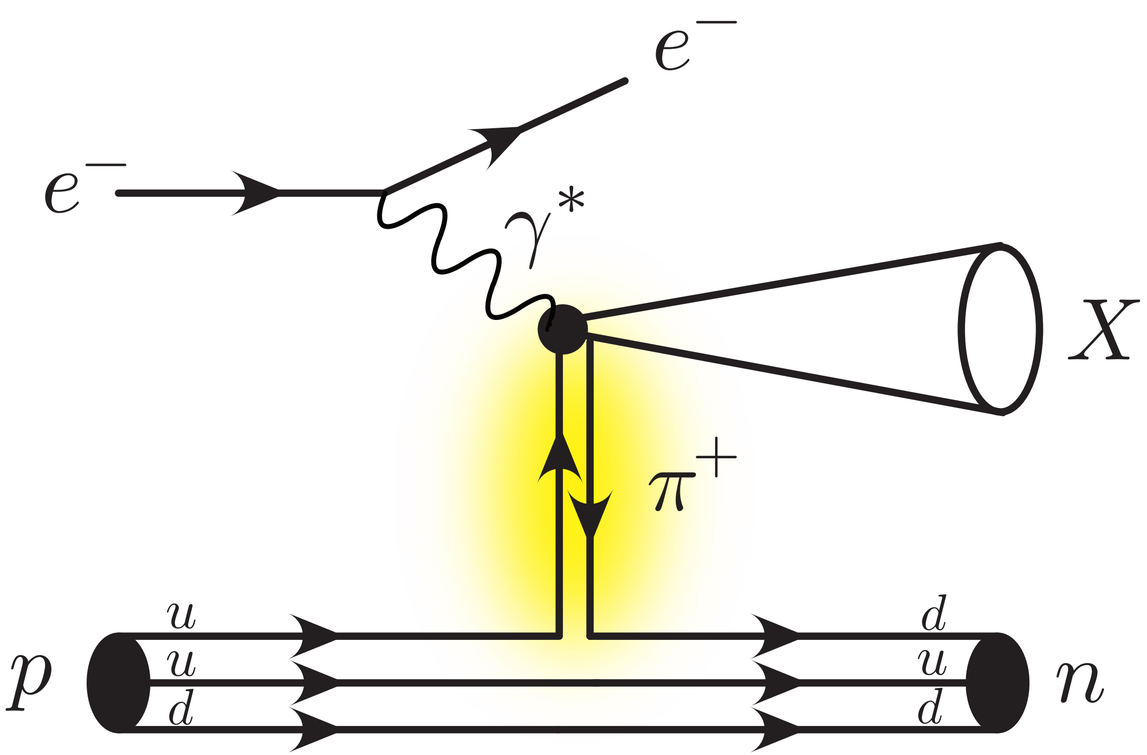
Naïve picture: it was proposed before QCD, yet people still use it!



PDFs: *longitudinal momentum only,*
GPDs: *longitudinal momentum + transverse position*
TMDs: *longitudinal momentum + transverse momentum*

(Figure: Argonne and Jefferson Lab)

First principles description?



Sullivan process: Belief

But! It…

- *is forward*
- *has a rapidity gap*

Forward gapped processes = **diffractive**

not known how to do from first principles until last year (2025)

(Lee, Schindler, Stewart)



Paper in progress with Stella T. Schindler, LANL P-3

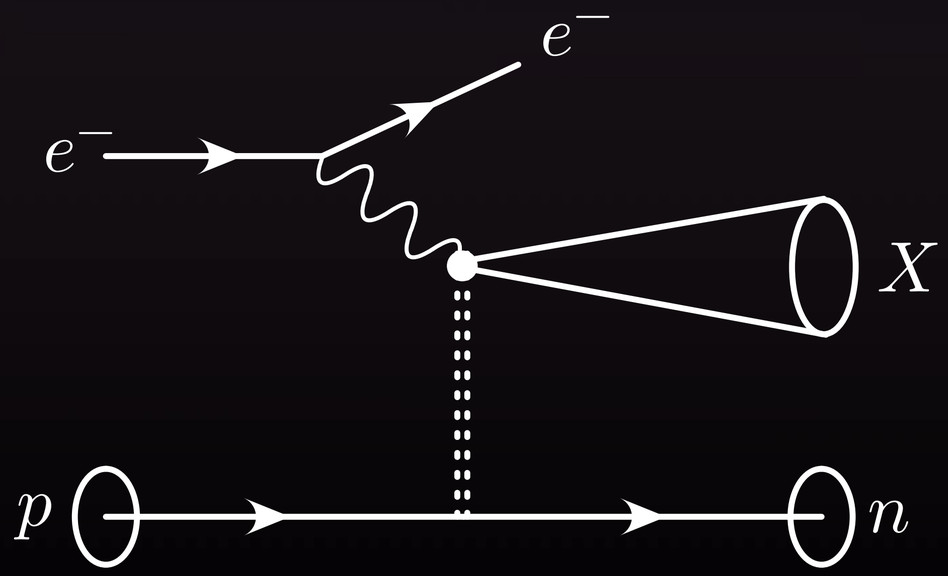
PART I.

What Sullivan process is(n't)

PART II. *EFT for diffraction*

PART III. *Towards predictions*

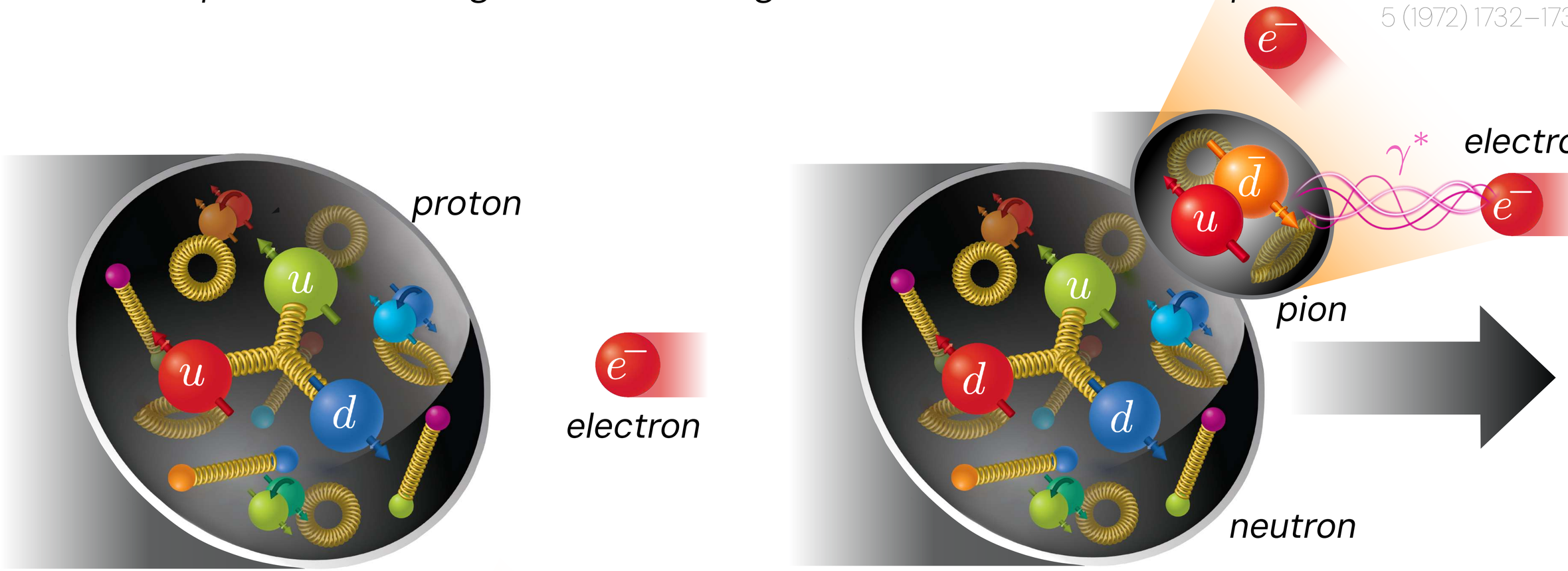
PART IV. *Extensions*



The Sullivan process: history

$ep \to eXn$

Proposed in 1971 right after learning of inner structure of the proton (Sullivan, Phys. Rev. D 5 (1972) 1732–1737)

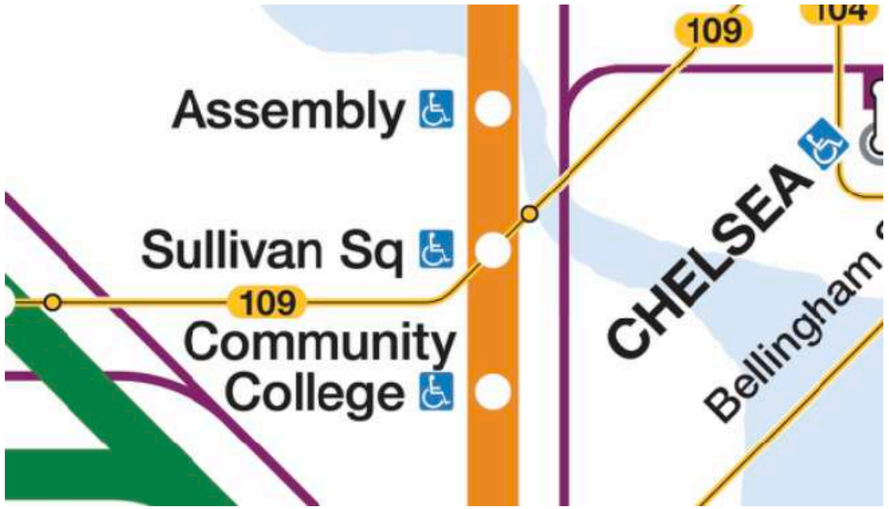


Idea:
only one pion exchanged

Claim:
able to access the internal gluon structure of the pion

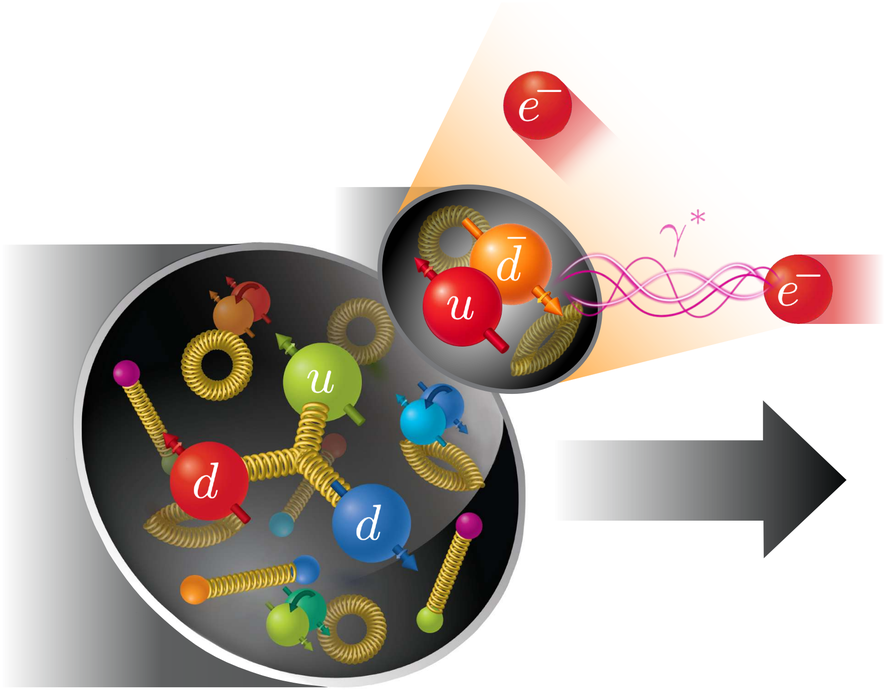
(Figure: BNL)

The Sullivan process: history



James Sullivan (governor)

Article Talk



James D. Sullivan (Boston U.)

astro-ph hep-ex nucl-ex



James P. Sullivan

"Hey, may the best monster win!"

—Sulley to Randall Boggs

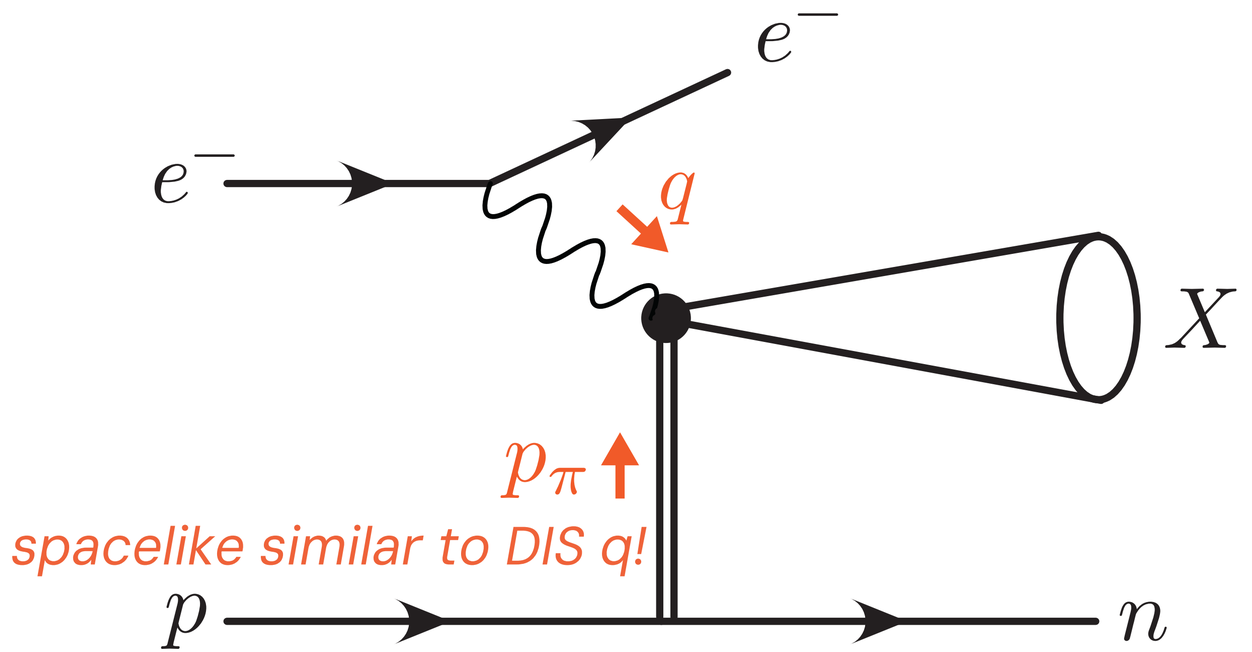
The Sullivan process: history

$ep \to eXn$

Idea:
only one pion exchanged

Why not (from logic):

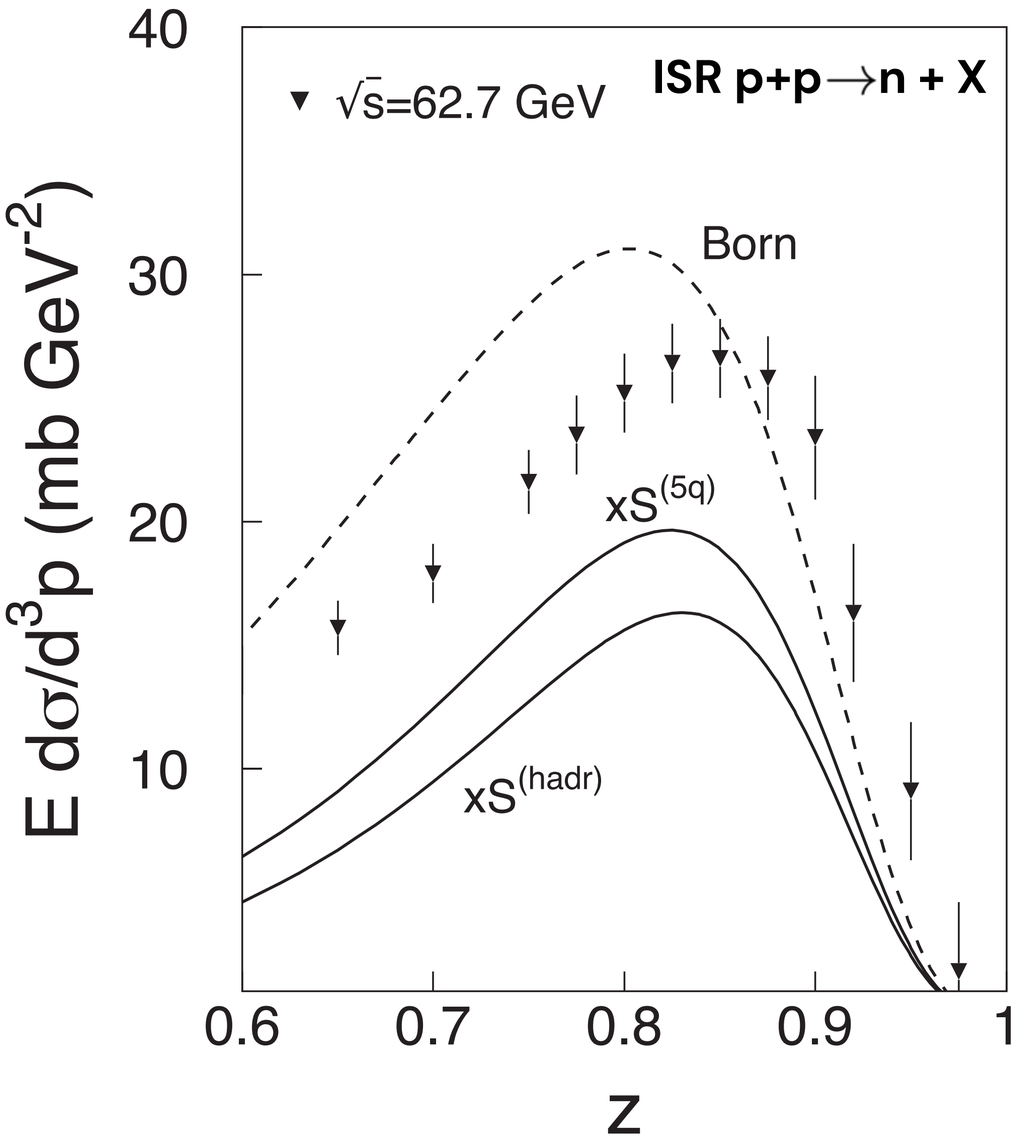
- intermediate particle is highly **off-shell**
$$p_\pi^2 < 0 \neq m_\pi^2 > 0$$
- $u\bar{d}$ quantum numbers $\neq$ a pion
- is not necessarily **a bound state hadron**



(Figure: BNL)

The Sullivan process: history

$ep \to eXn$



Experiments disagree!

Why not (from experiments):
Pion **overpredicts** forward neutrons measurements for cross section

Born: pion exchange

xS: cross section with different types of absorptive corrections

(Kopeliovich, Potashnikova, Schmidt, Soffer 0805.4534, 1109.2500)

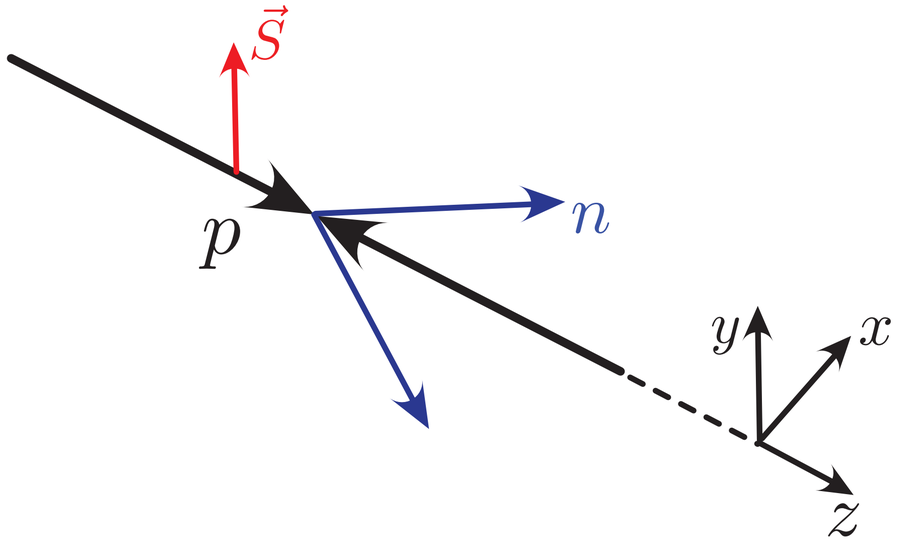
The Sullivan process: history

$ep \to eXn$

Why not (from experiments):

Pion does not **create enough spin asymmetry.**

People try to model difference with absorptive corrections or extra resonances.

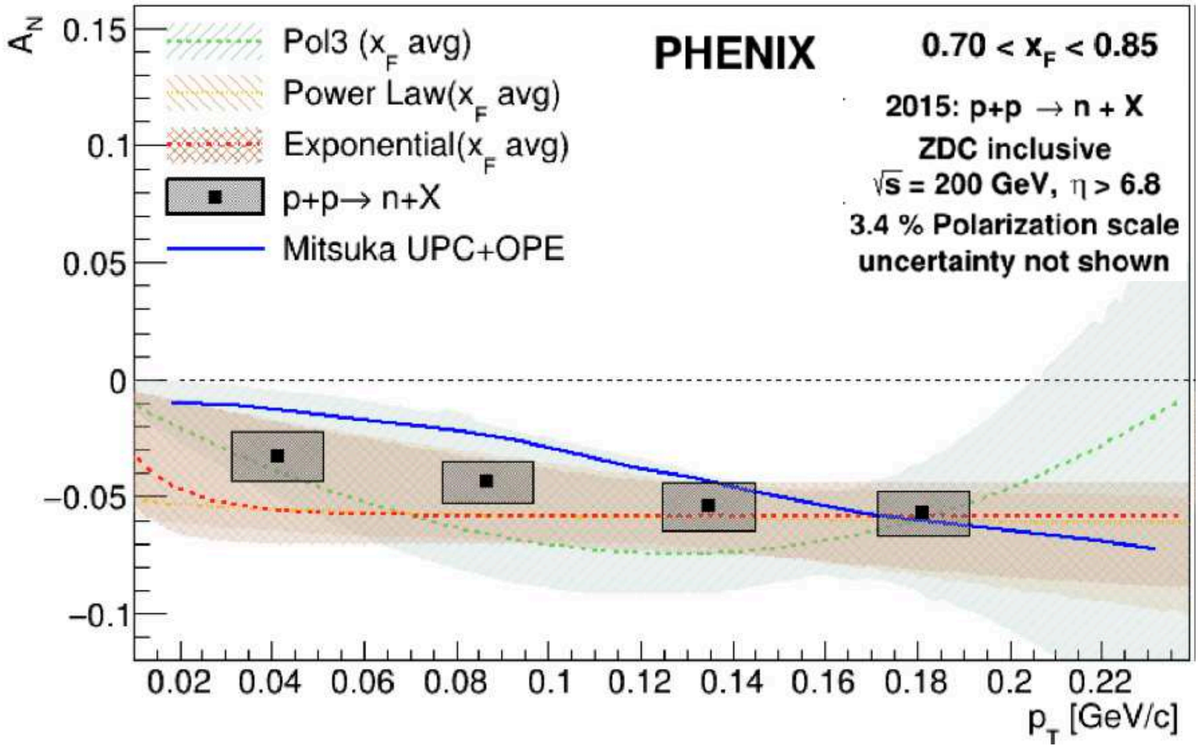


Spin asymmetry: $A_N \propto \dfrac{N_L - N_R}{N_L + N_R}$

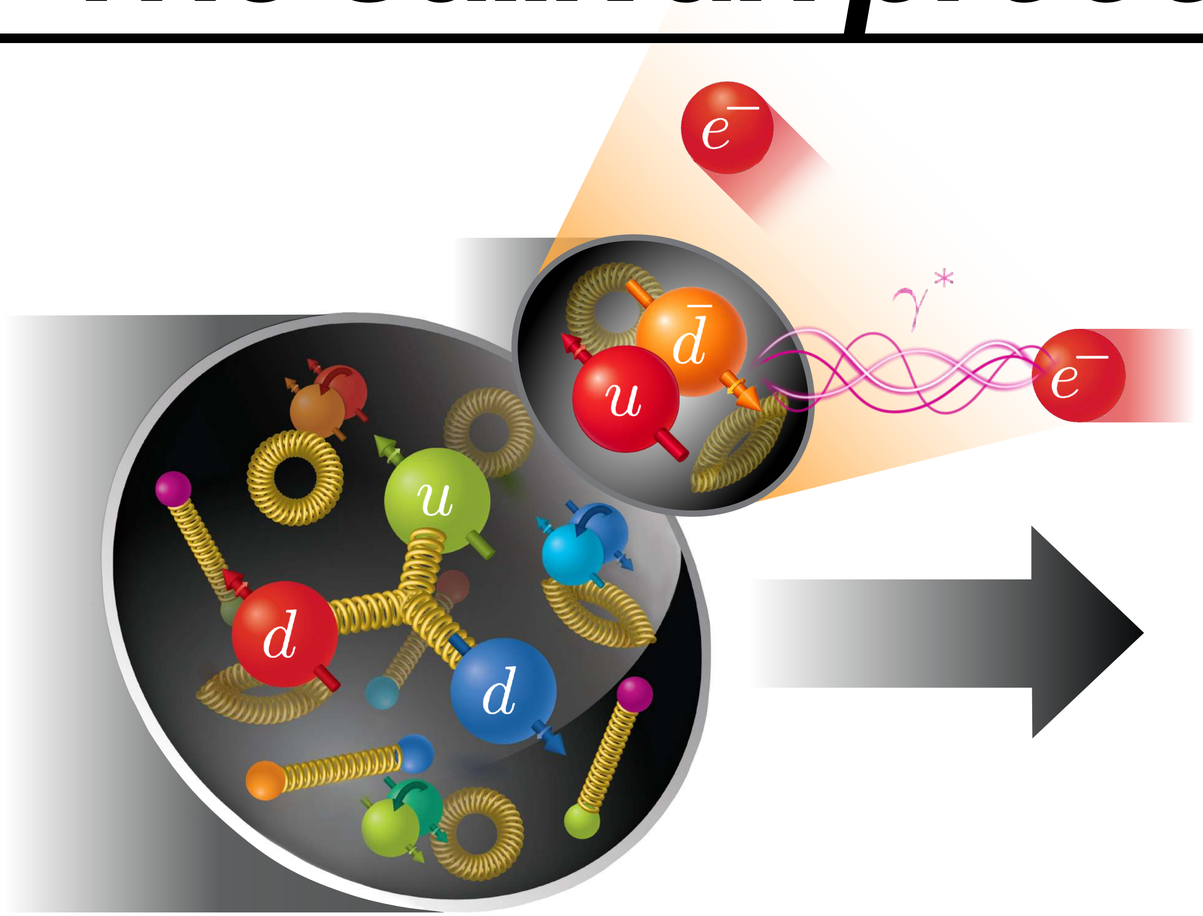
(PHENIX 2110.07504)

Pion model: $A_N = 0$

Experiment:

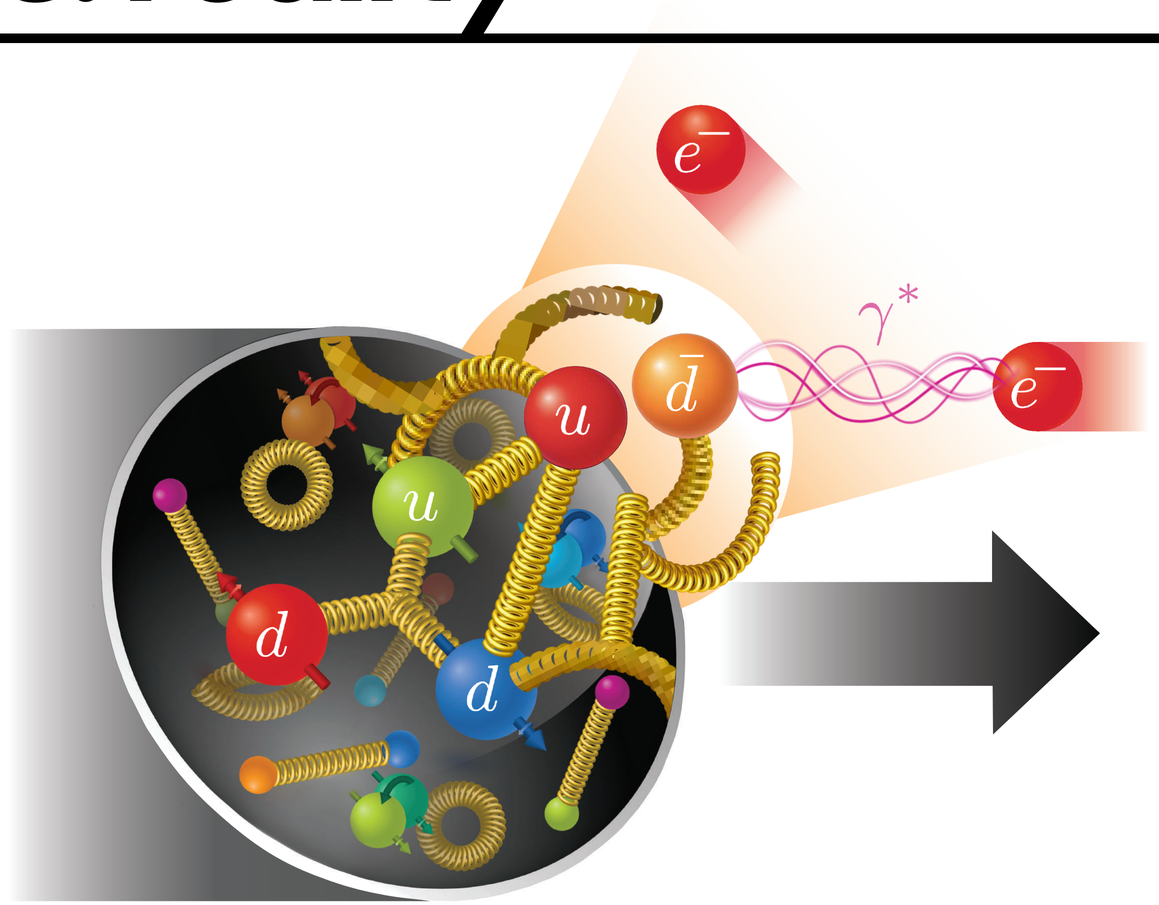


The Sullivan process: reality



History

- Mediated by a pion
- Can access pion's GPDs
- But models don't fit data

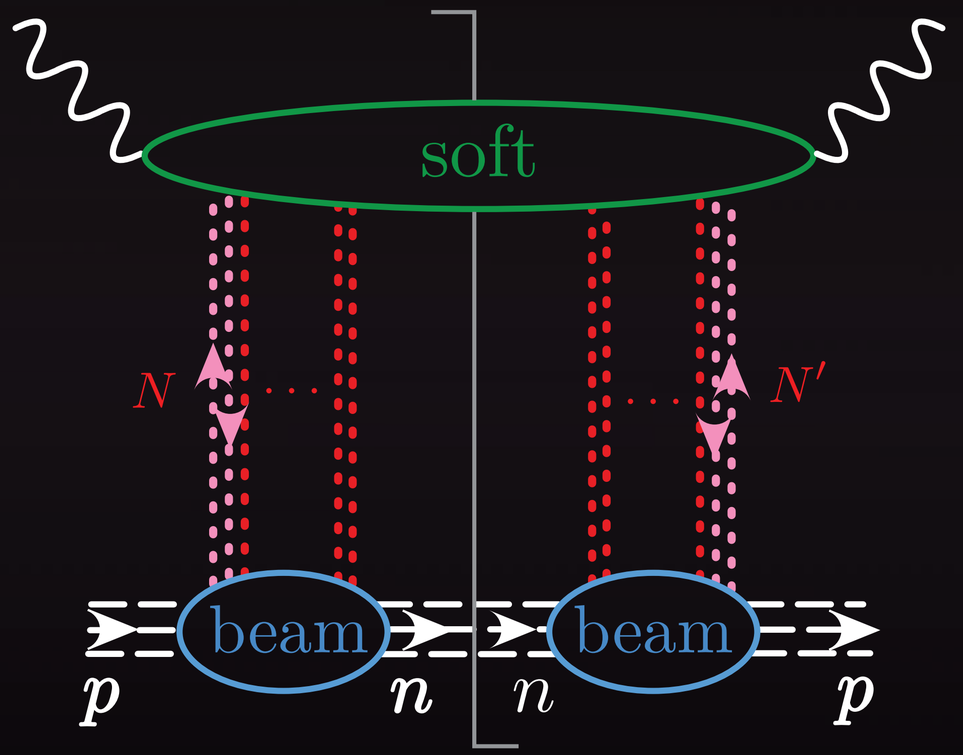


Modernity

- Mediated by a sea of gluons and two quarks
- Not even close to GPDs

PART I. *What Sullivan process is(n't)*

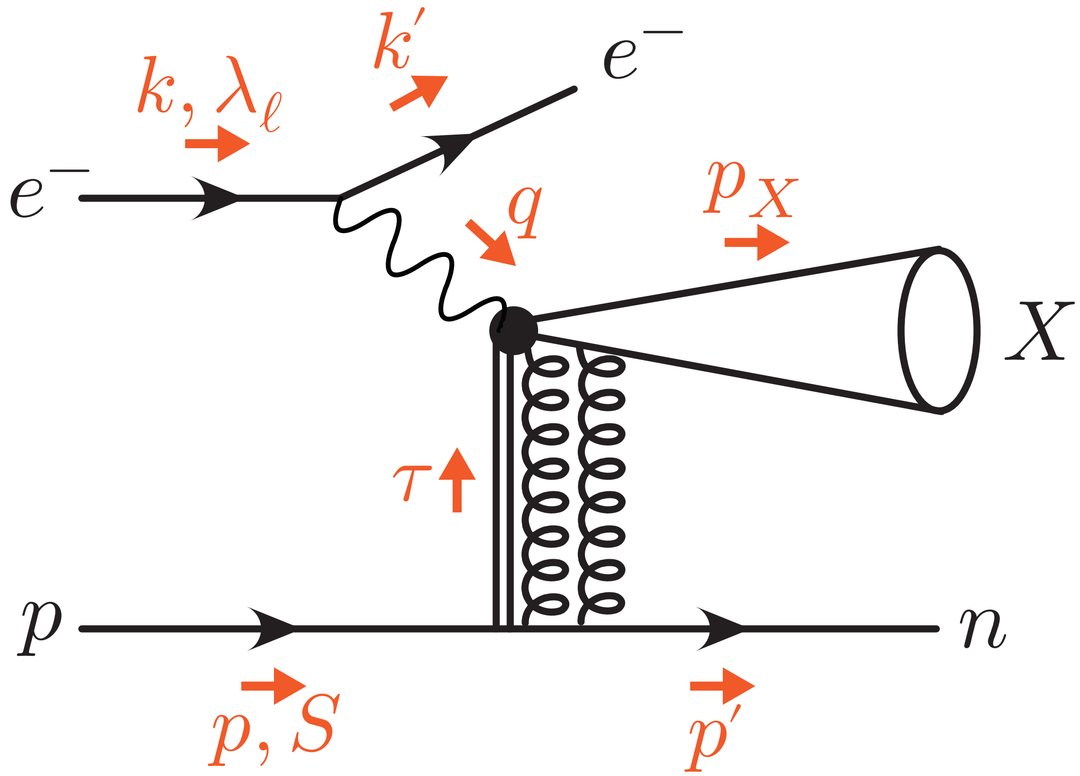
PART II.
EFT for diffraction



PART III. *Towards predictions*

PART IV. *Extensions*

Kinematic variables for Sullivan

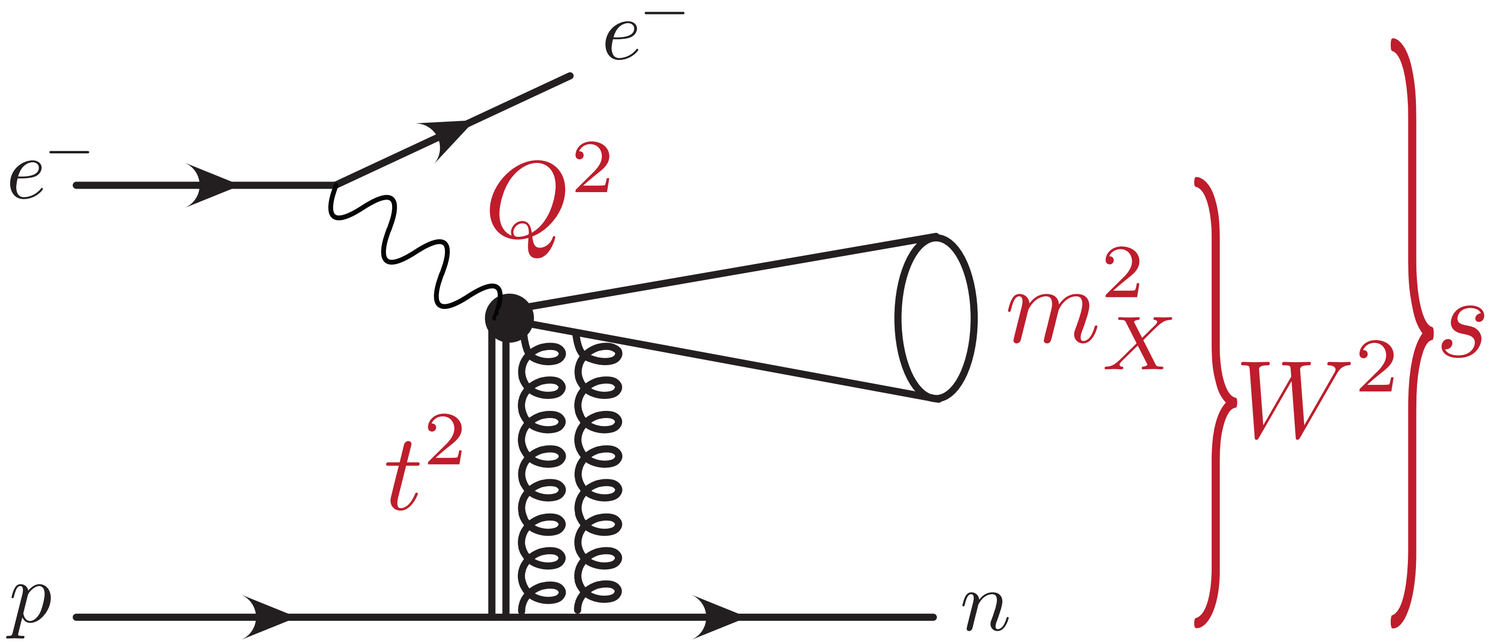


Momentum fractions			
$x = \dfrac{Q^2}{2p \cdot q}$	$\beta = \dfrac{Q^2}{2q \cdot \tau}$	$y = \dfrac{p \cdot q}{p \cdot k}$	$\bar{x} = \dfrac{k \cdot \tau}{k \cdot p}$

$$W^2 \equiv (p+q)^2$$

$$s = (p+k)^2$$

Kinematic variables for Sullivan



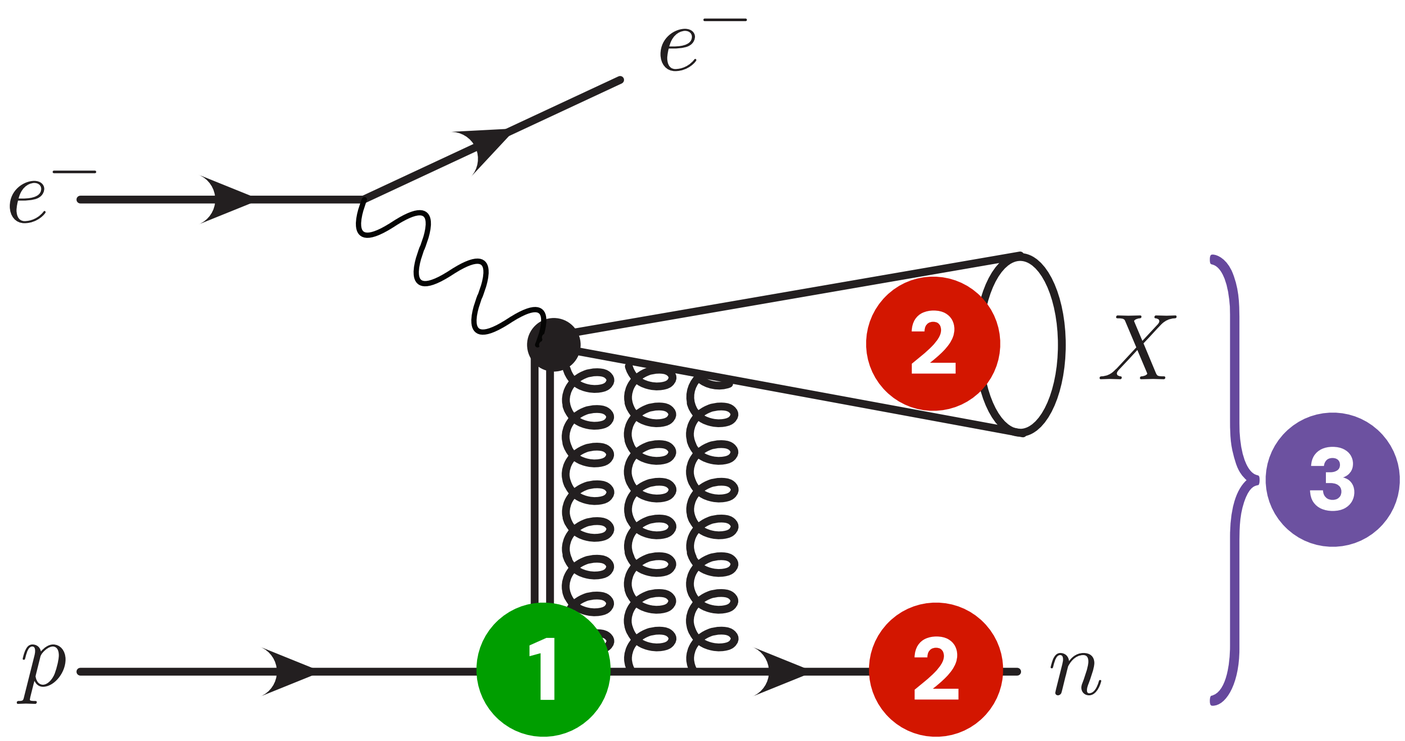
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$$W^2 \equiv (p+q)^2$$

$$s = (p+k)^2$$

Diffractive process: kinematic reqs.

*Not dealing with the leptonic part



1. High energy, forward scattering:

high energy $x \ll 1$,

small mom. transfer $-t \ll W^2$

2. Clear separation: *small invariant mass of the final states*

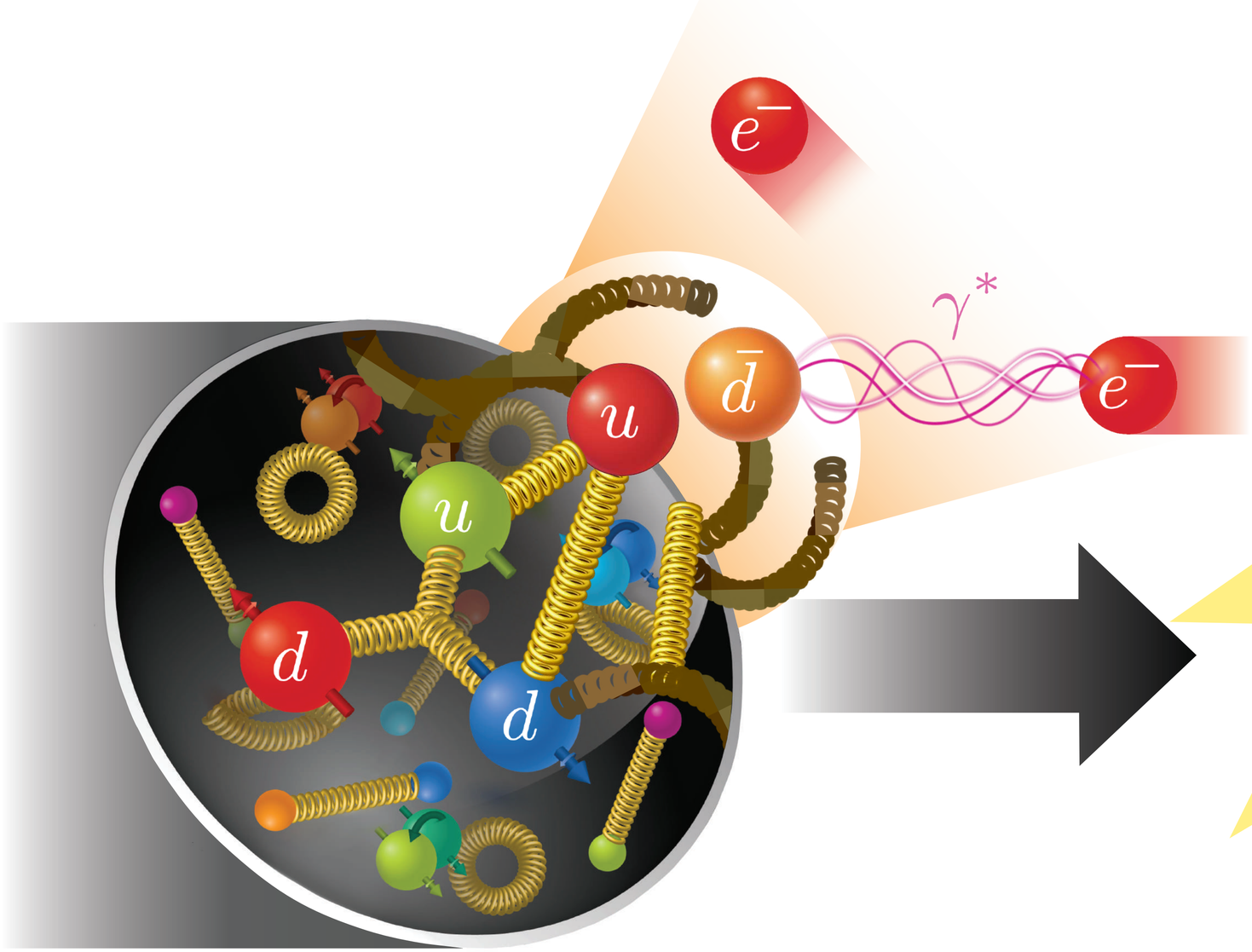
$$\Lambda^2_{\mathrm{QCD}} \lesssim m_X^2 \ll W^2,$$
$$\Lambda^2_{\mathrm{QCD}} \sim m_n^2 \ll W^2$$

3. Rapidity gap:

$$\ln \frac{p_X^-}{p_X^+} \ll \ln \frac{p'^-}{p'^+}$$

Small-x Sullivan process is always diffractive

The Sullivan process: correct framework



Right framework:
Soft-Collinear Effective Field Theory (SCET) for diffraction

(Lee, Schindler, Stewart)

Effective field theory in a nutshell

Full QCD

All details included



What the detector resolves

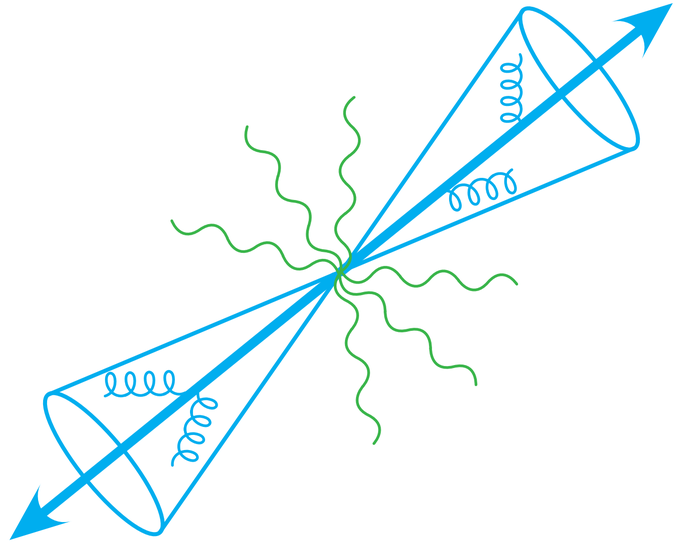
energy flow, jets, event shapes, vetoed radiation



NOT:
microscopic high-virtuality history of the collision

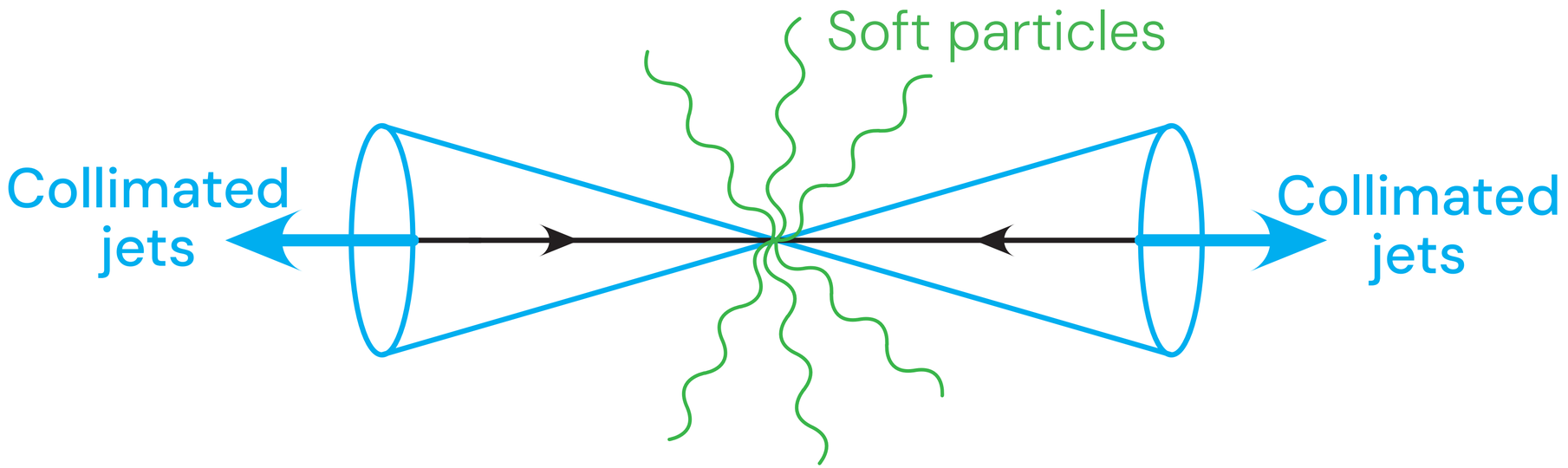
Effective theory

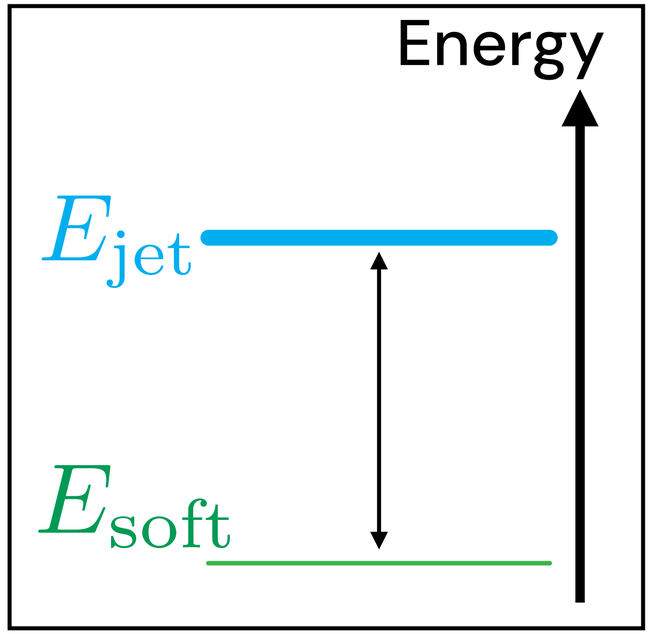
Only relevant modes



Keep modes that can change the measured observable

Effective field theory in a nutshell





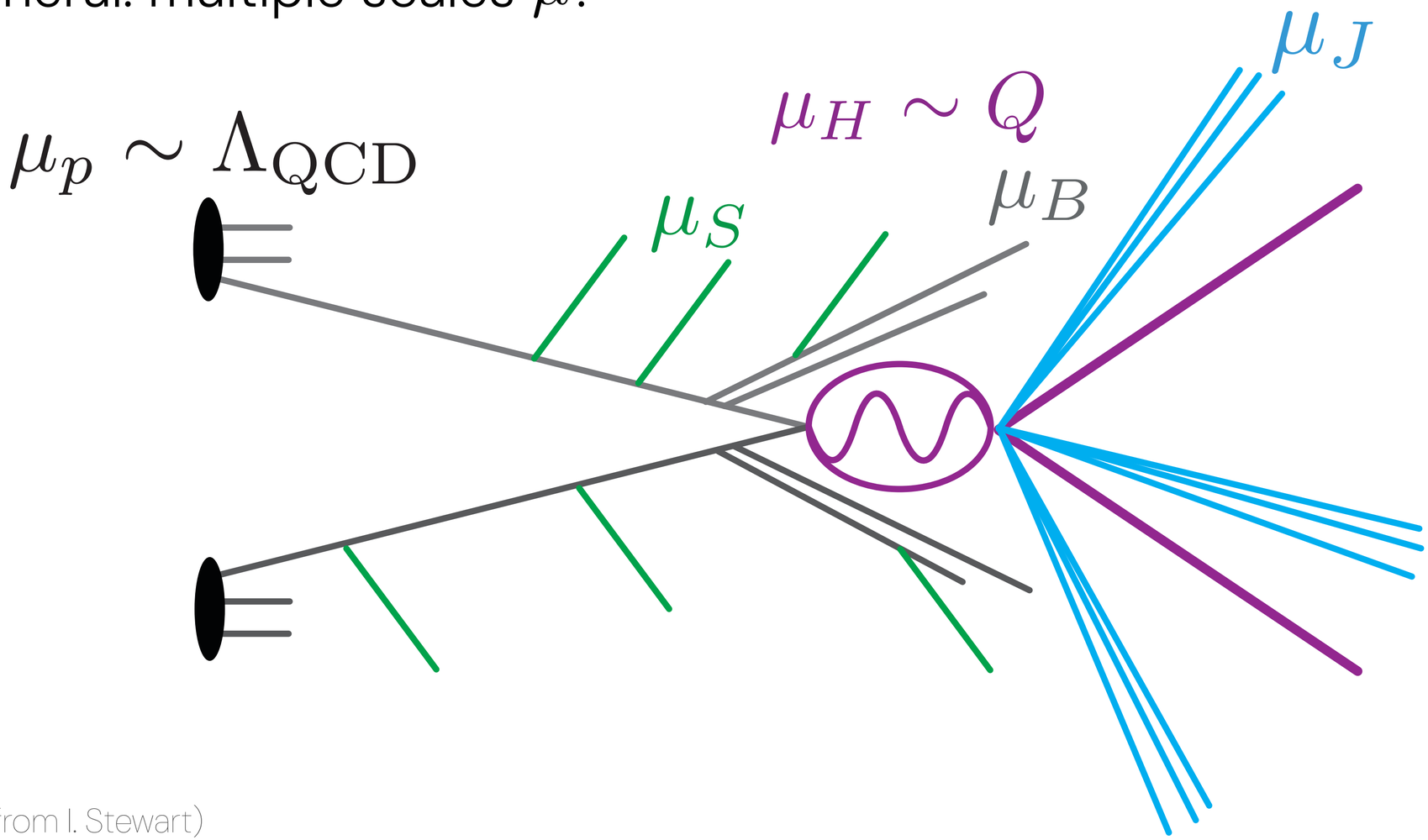
Lagrangian: $\mathcal{L}_{\rm QCD} \xrightarrow[\text{irrelevant modes}]{\text{integrate out}} \mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm jet} + \mathcal{L}_{\rm soft}$

Observables: $\langle \mathcal{O} \rangle \to \langle \mathcal{O}_{\rm jet} \rangle \times \langle \mathcal{O}_{\rm soft} \rangle \times \langle \mathcal{O}_{\rm jet} \rangle$

(Figure adapted from I. Stewart. See e.g. 2303.02579)

Effective field theory in a nutshell

- Systematically improvable
- Simplifies study of: factorization, perturbative corrections…
- In general: multiple scales μ!

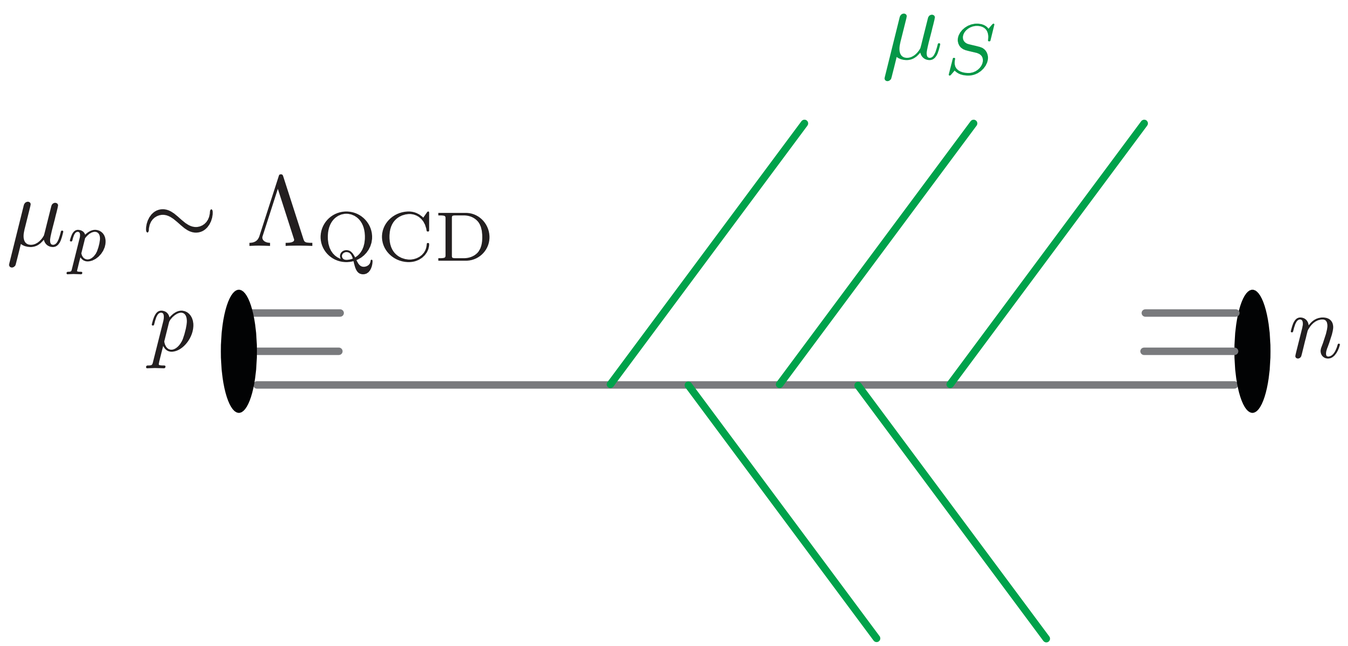


(Figure adapted from I. Stewart)

SCET Lagrangian

*Soft-Collinear Effective Field Theory

$$\lambda = \frac{Q}{\sqrt{s}} \ll 1$$



Relevant modes

$(p^+, p^-, p_\perp)$

Collinear	$\sqrt{s}(\lambda^2, 1, \lambda)$
Soft	$\sqrt{s}(\lambda, \lambda, \lambda)$

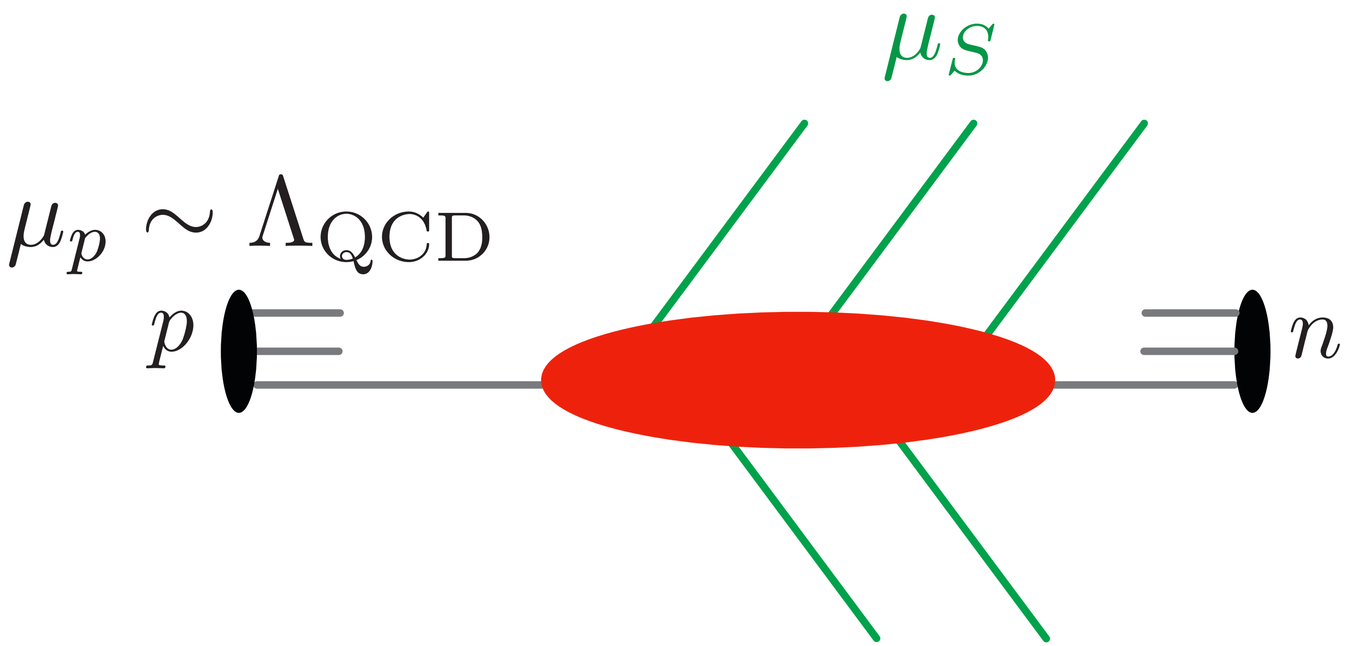
$$\mathcal{L}^{(0)}_{\rm SCET} \supset \mathcal{L}_{\rm collinear} + \mathcal{L}_{\rm soft}$$

(Rothstein and Stewart, 1601.04695)

SCET Lagrangian

*Soft-Collinear Effective Field Theory

$$\lambda = \frac{Q}{\sqrt{s}} \ll 1$$



Relevant modes

$$(p^+, p^-, p_\perp)$$

Collinear	$\sqrt{s}(\lambda^2, 1, \lambda)$
Soft	$\sqrt{s}(\lambda, \lambda, \lambda)$
Glauber	$\sqrt{s}(\lambda^2, \lambda, \lambda)$

Connects the transverse momentum without affecting the other components

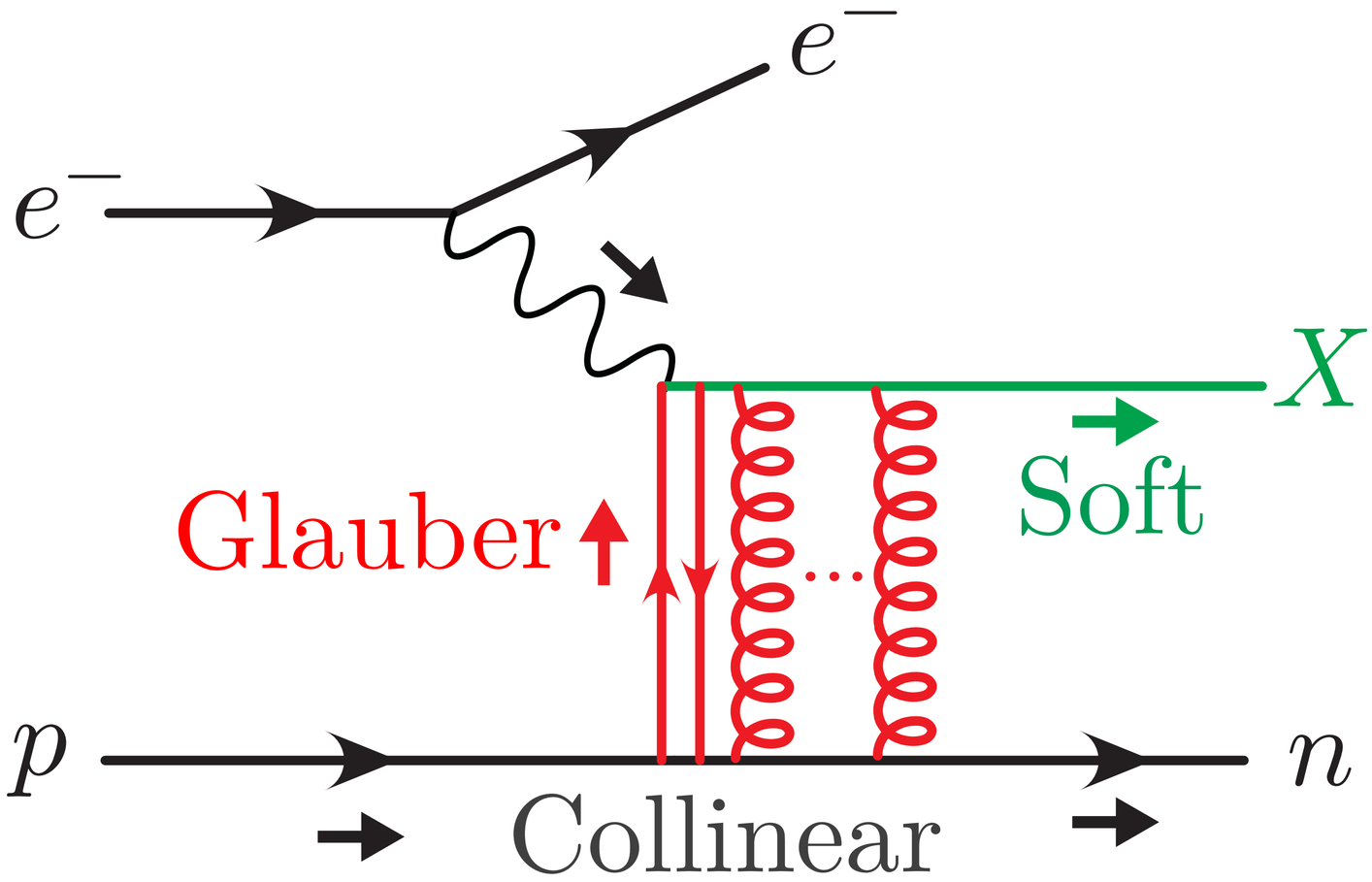
$$\mathcal{L}^{(0)}_{\rm SCET} \supset \mathcal{L}_{\rm collinear} + \mathcal{L}_{\rm soft} + \mathcal{L}_{\rm Glauber}$$



@ Quantum Computing building at Harvard

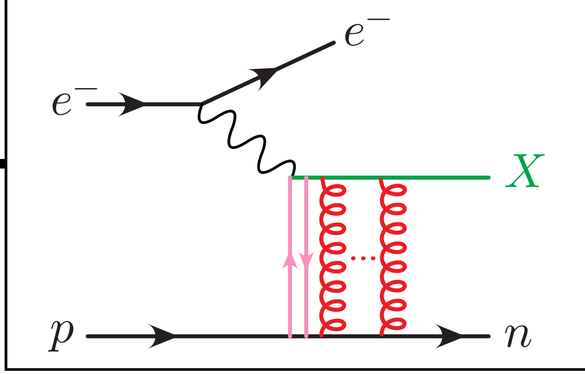
(Rothstein and Stewart, 1601.04695)

SCET modes in Sullivan



SCET is a right fit for the diffractive Sullivan process!

Glauber Lagrangian



'Connections' between sectors

Leading

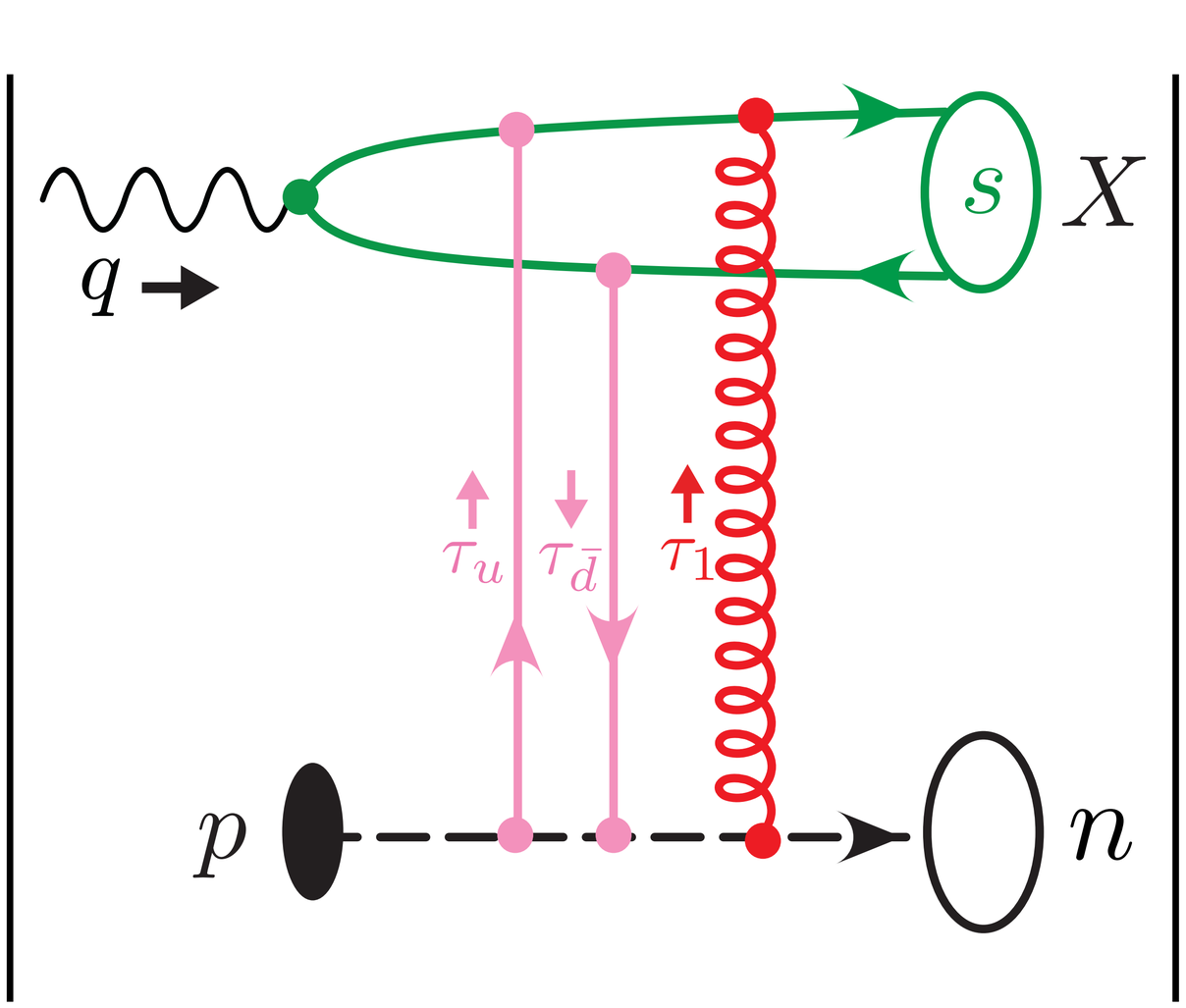
$$\mathcal{L}^{(0)}_{\text{Glauber}} \supset \mathcal{O}^B_s \left(\frac{1}{\tau_\perp^2}\right) \mathcal{O}^B_n \longrightarrow \mathcal{O}^B_n \frac{1}{\tau_\perp^2} \mathcal{O}^B_s$$

Sub-leading

$$\mathcal{L}^{(1/2)}_{\text{Glauber}} \supset \mathcal{O}^n_s \left(\frac{\not{n}_\perp}{\tau_\perp^2}\right) \bar{\mathcal{O}}_n \longrightarrow \bar{\mathcal{O}}_n \frac{\not{n}_\perp}{\tau_\perp^2} \mathcal{O}^n_s$$

(Rothstein, Stewart, 1601.04695v3)
(Moult, Solon,, Stewart, Vita,, 1709.09174v1)

Factorization



$$= \left(\frac{1}{\not{\tau}_{u\perp}} \frac{1}{\not{\tau}_{\bar{d}\perp}} \right)^2 \left(\frac{1}{\tau_{1\perp}^2} \right)^2$$

Soft

$$|\langle 0|J^\mu \mathcal{O}_s^n \bar{\mathcal{O}}_s^n \mathcal{O}_s^B|X\rangle|^2$$

$\otimes_\perp$

$\otimes_\perp$

Jets

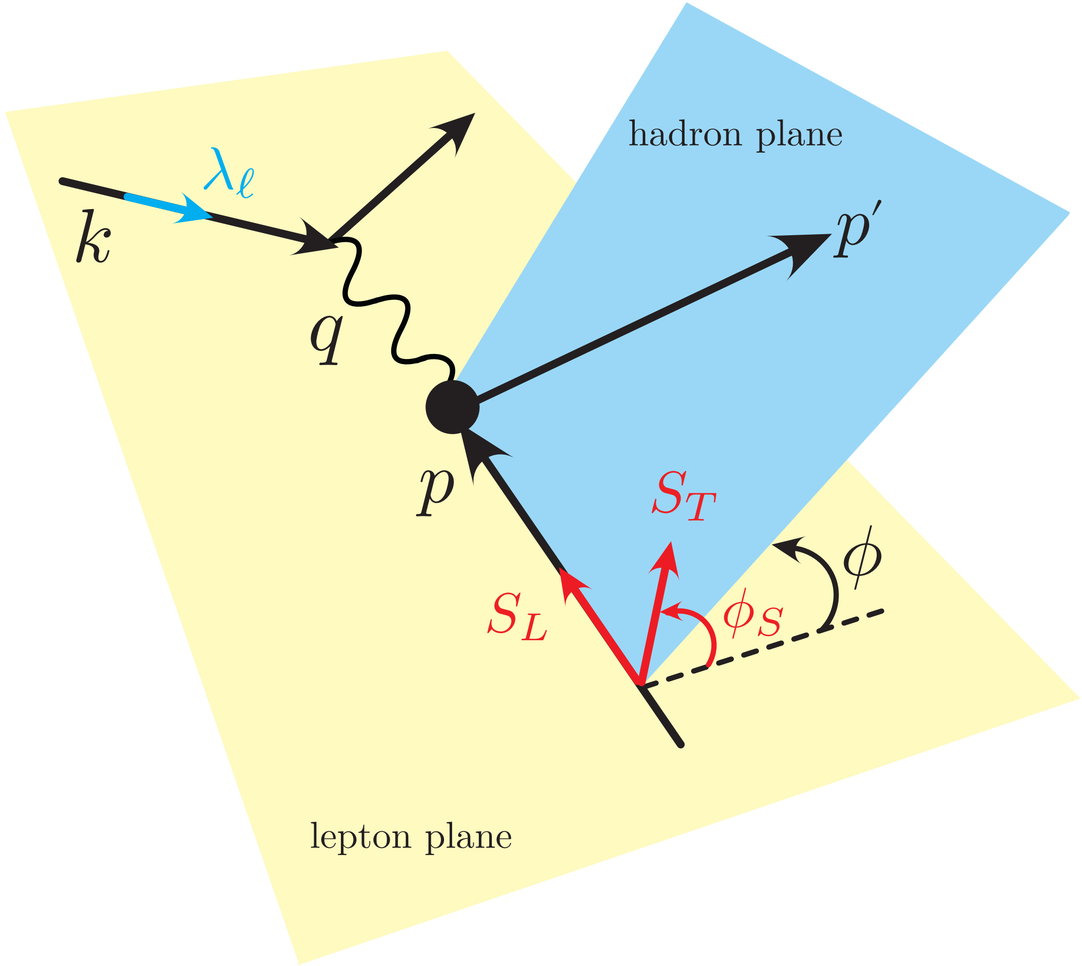
$$|\langle p|\bar{\mathcal{O}}_n \mathcal{O}_n \mathcal{O}_n^B|n\rangle|^2$$

Diffractive Structure Functions

Can use SCET to derive the expressions for structure functions

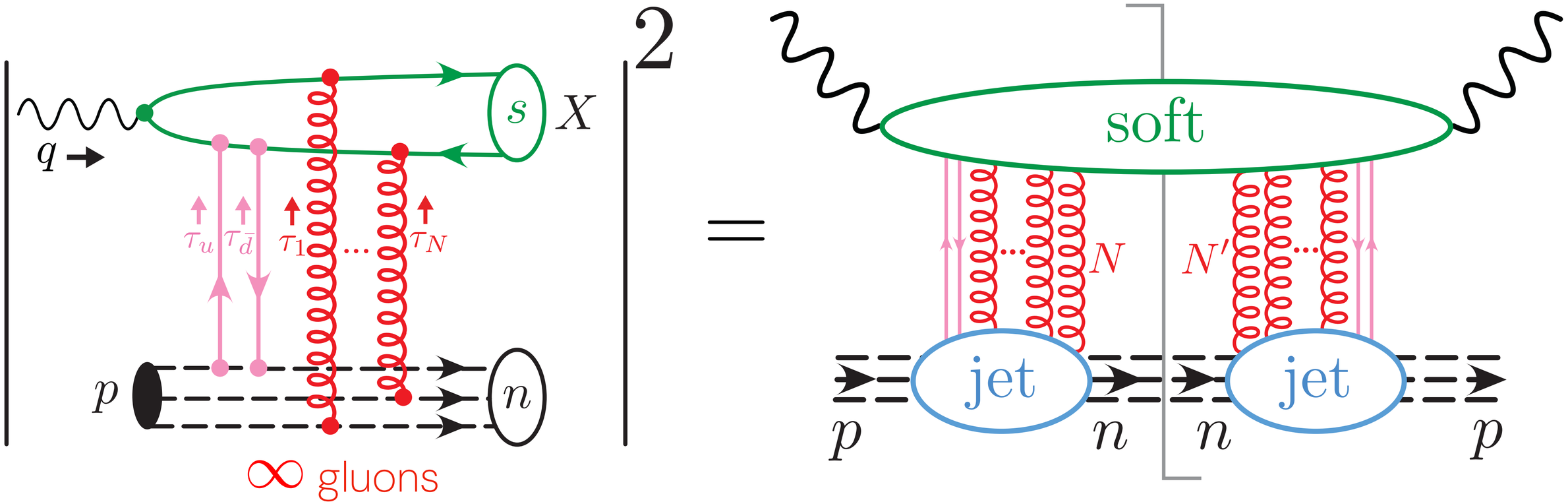
$$\frac{d^5\sigma}{dx\,dQ^2\,d\beta\,dt\,d\phi} = \mathcal{N}\Big\{$$

$$\begin{aligned}
& F_L^D,\ F_2^D,\ F_3^D,\ F_4^D && \times (y,\phi) && \text{UU}\\
+\ & \lambda_\ell \left[F_{4A}^D\right] && \times (y,\phi) && \text{LU}\\
+\ & S_L \left[F_5^D,\ F_7^D\right] && \times (y,\phi) && \text{UL}\\
+\ & S_T \left[F_{LP}^D,\ F_{2P}^D,\ F_{3P}^D,\ F_{4P}^D,\ F_5^D,\ F_6^D,\ F_7^D,\ F_8^D\right] && \times (y,\phi,\phi_S) && \text{UT}\\
+\ & \lambda_\ell S_L \left[F_{5A}^D,\ F_{7A}^D\right] && \times (y,\phi) && \text{LL}\\
+\ & \lambda_\ell S_T \left[F_{6A}^D,\ F_{7A}^D,\ F_{8A}^D,\ F_{4AP}^D\right] && \times (y,\phi,\phi_S) && \text{LT}
\end{aligned}$$



Diffractive process: 18 structure functions

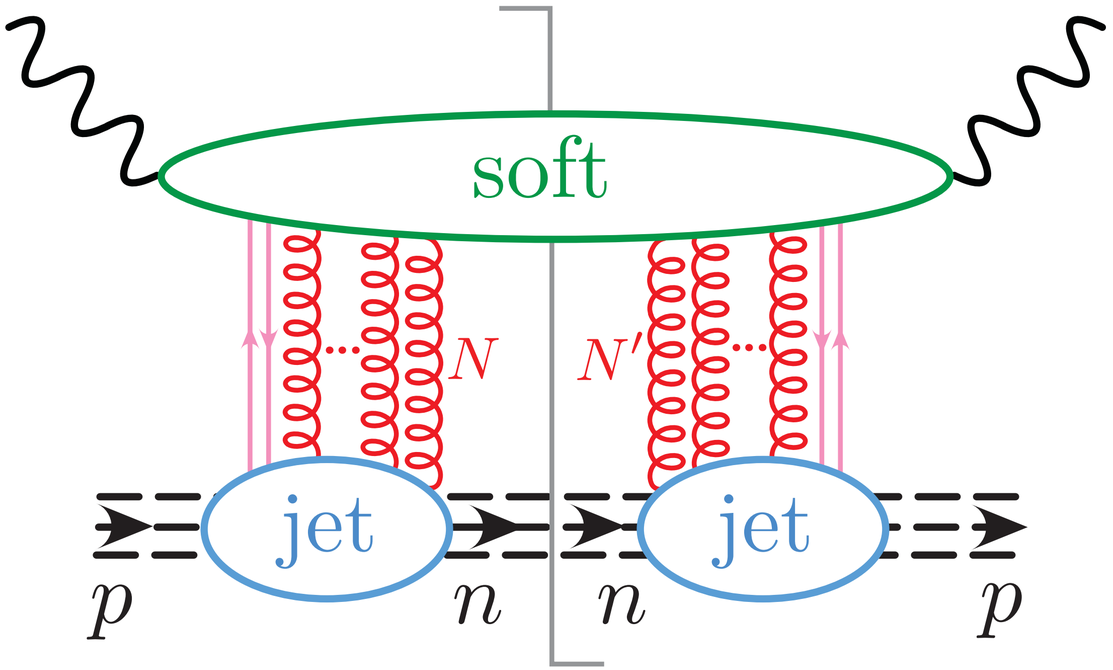
Factorization of the Sullivan process



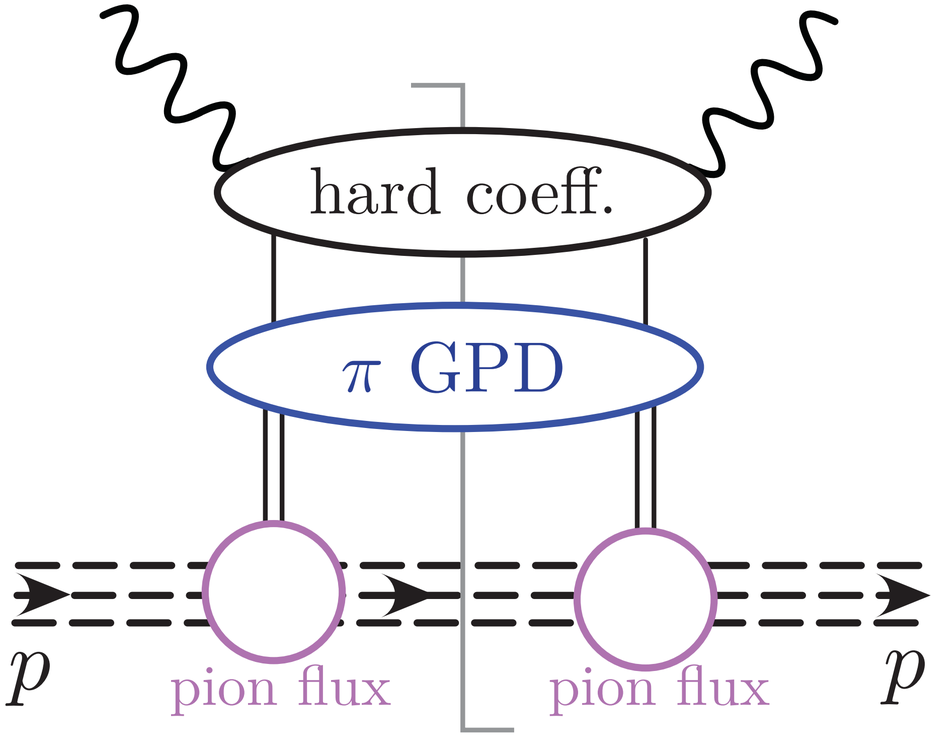
$$F_i^D \propto \sum_{N,N'=1}^{\infty} \iint_{NN'}^{\perp u\bar{d}} S_{i\,NN'}(\dots) \times J_N(\dots) \times J_{N'}(\dots)\,.$$

Glaubers Soft Jet Jet

Glauber vs. pion exchange



$$F_i^D \sim S \otimes_\perp J^2$$



$$F_i \sim (f^2_{\pi/p}) \otimes [H \otimes f_{i/\pi}]$$

Completely different operator structure and physics!

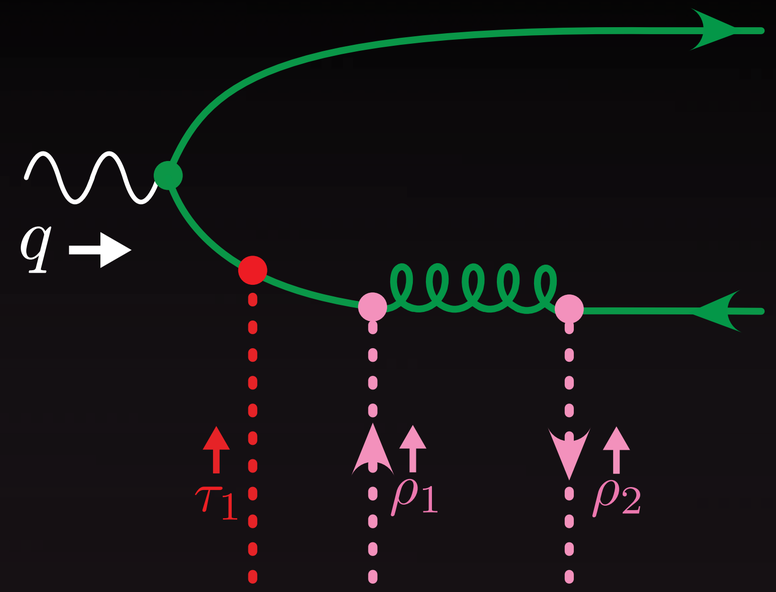
(H1 Collaboration, 1001.0532v1)

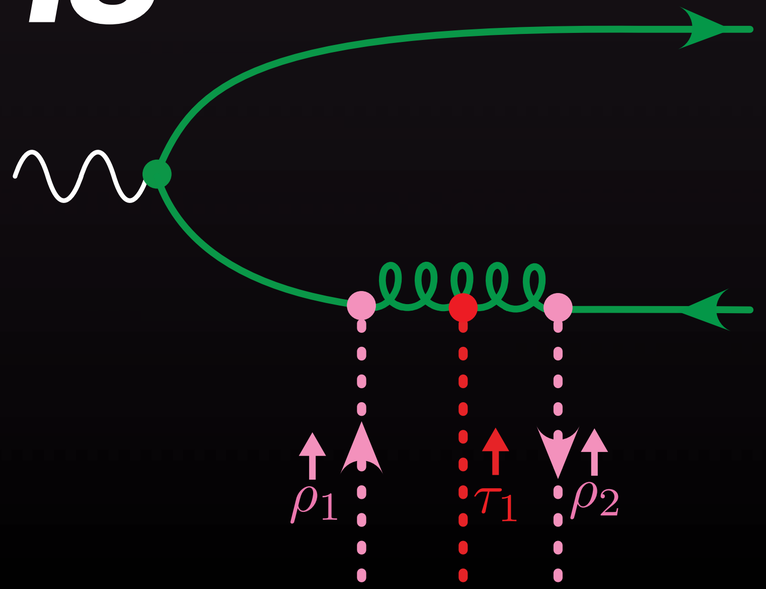
PART I. *What Sullivan process is(n't)*

PART II. *EFT for diffraction*

PART III. *Towards predictions*

PART IV. *Extensions*





Towards predictions

What do we need for full predictions?

$$F_i^D = S_{iNN'} \otimes_\perp (J_N \times J_{N'})$$

- cannot perturbatively calculate $\rightarrow$ *NP effects in protons*

- cannot use lattice calculations
- very hard to extract from experiment

$\rightarrow$ *difficult to deal with Glaubers connecting the soft and collinear modes*

- cannot take ratios $\rightarrow$ *have non-trivial convolution (at least two quarks present = at least one integral over the momenta)*

Towards predictions

What do we need for full predictions?

$$F_i^D = S_{iNN'} \otimes_\perp (J_N \times J_{N'})$$

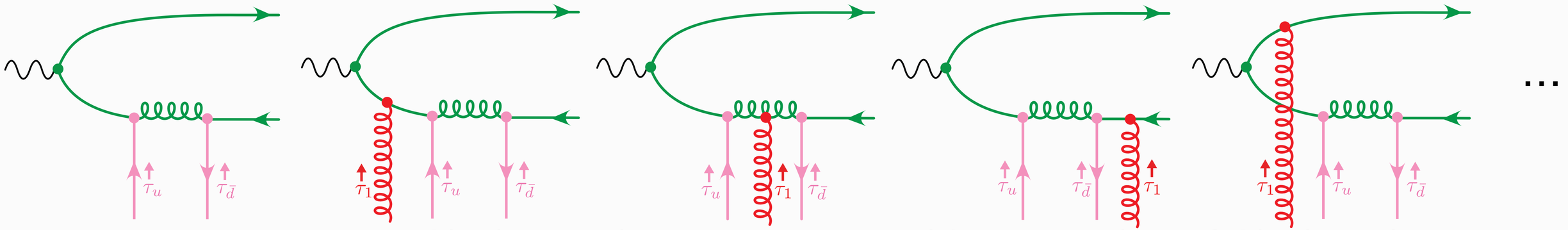
- can estimate structure function sizes
- can do positivity/consistency checks for structure functions
- can model jet function to approximate the cross sections

Towards predictions

What do we need for full predictions?

$$F_i^D = S_{iNN'} \otimes_\perp (J_N \times J_{N'})$$

Difficult, but possible (perturbative) calculations



Have to sum over Glauber exchanges up to an arbitrary number of Glauber gluons

low energy Glauber gluons are all of the same order

$$-\sum_i \tau_i^2 = -t \gg \Lambda_{\rm QCD}$$

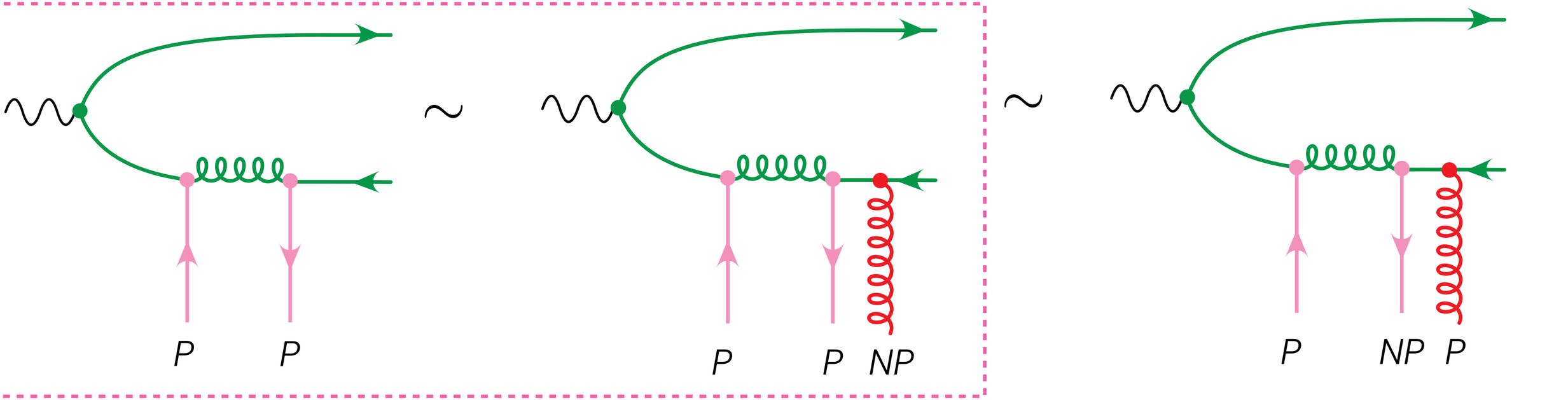
Mapping onto 2-quark exchange?

In the limit $(\tau_u + \tau_{\bar{d}} + \tau_1 + \cdots + \tau_N)^2 \gg \Lambda^2_{\rm QCD}$

total momentum transfer (perturbative)

$\tau^2 \sim \Lambda^2_{\rm QCD}$

(non-perturbative)



Same kinematic structure!

Same order in α_S, different kinematic structure!

Full Soft function $\neq S_{q\bar{q}} \times$ Color factors

Not a pion!

Structure functions

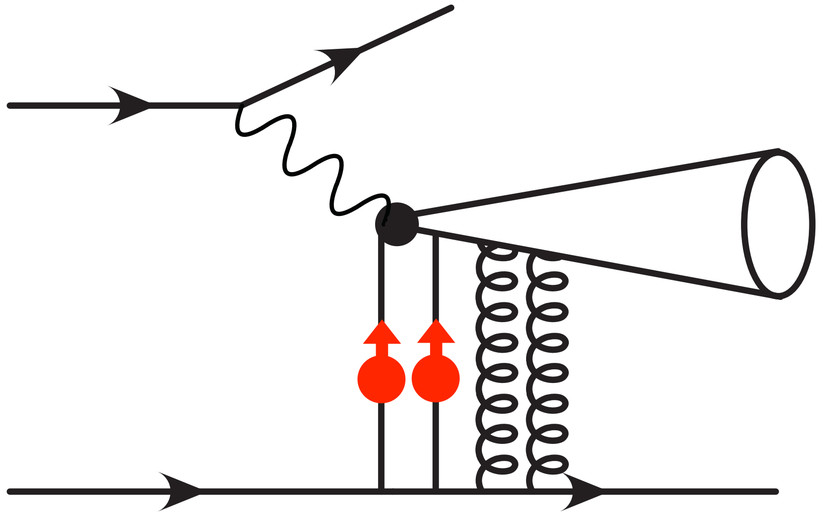
In diffraction: 18 structure functions F_i^D

$ep \to eXp$
10/18 are leading

$ep \to eXn$
18/18 are leading

Glauber fermions carry spin:

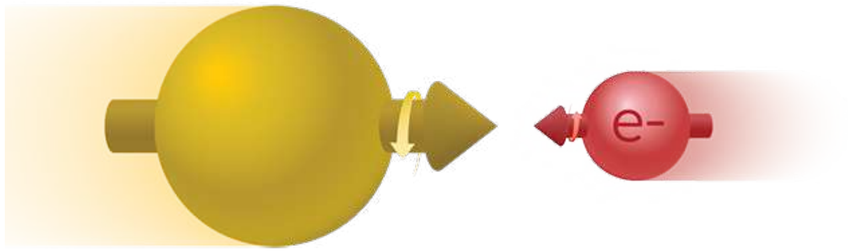
larger possible set of leading power structure functions



Spin asymmetries

In EIC both polarized e and p/A :

probing a larger set of asymmetries for the first time

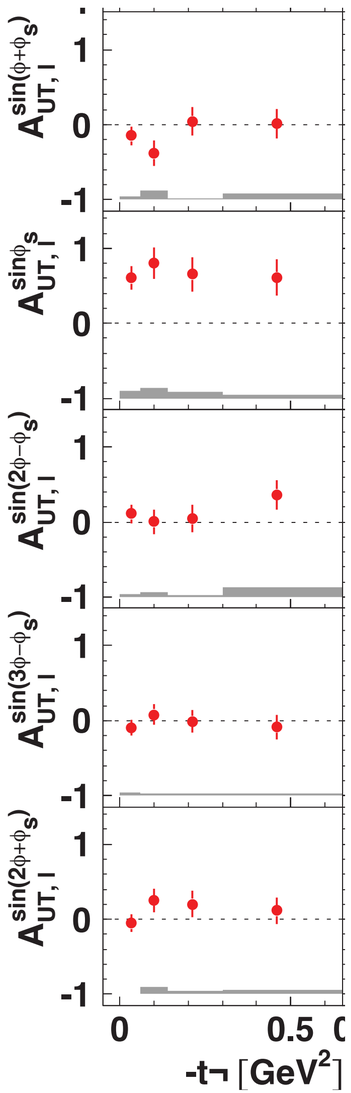


Asymmetry (beam/target)		Accessed structure functions
A_{LU}	→ ●	F_{4A}^D
A_{UL}	● →	F_5^D, F_7^D
A_{UT}	● ↑	$F_{LP}^D, F_{2P}^D, F_{3P}^D, F_{4P}^D, F_5^D, F_6^D, F_7^D, F_8^D$
A_{LL}	→ →	F_{5A}^D, F_{7A}^D
A_{LT}	→ ↑	$F_{4AP}^D, F_{5A}^D, F_{6A}^D, F_{7A}^D, F_{8A}^D$

● unpolarized → longitudinally polarized ↑ transversally polarized

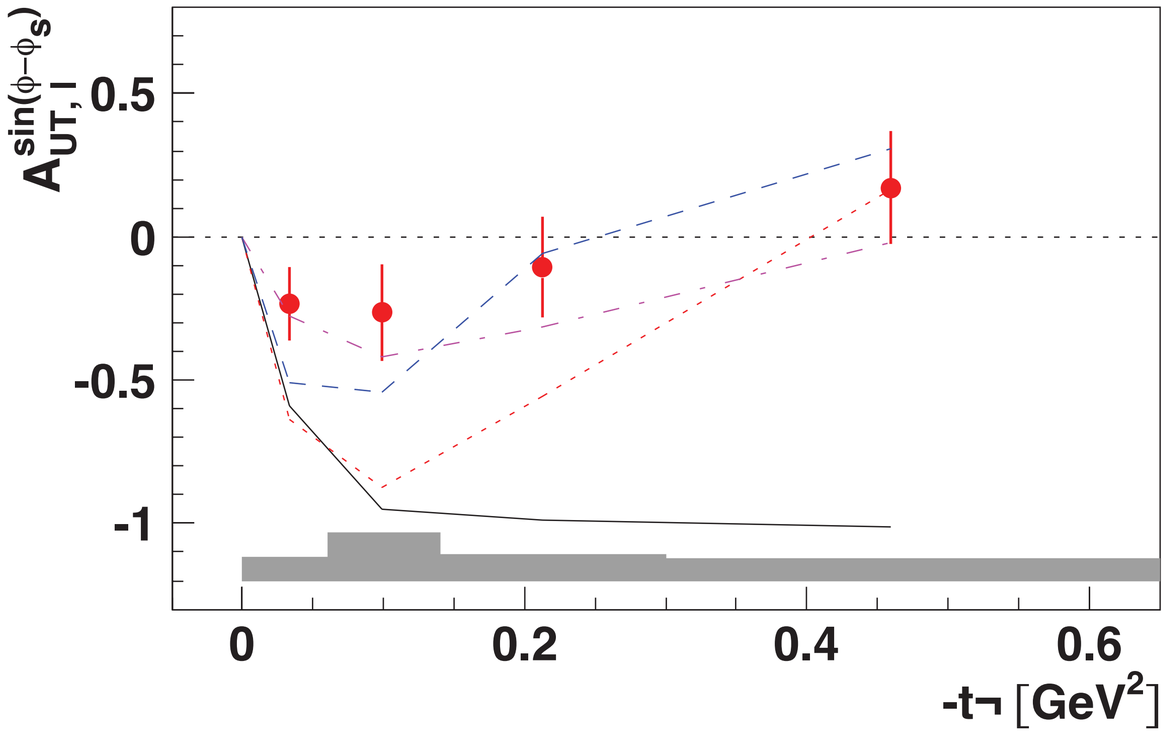
Spin asymmetries

Measured asymmetry



(HERMES 0907.2596v1)

Example: $A_{UT,\ell}^{\sin(\phi-\phi_S)}$



$$A_{UT,\ell}^{\sin(\phi-\phi_S)} = \frac{\cos\theta\left[F_{2P}^D - F_{LP}^D + F_8^D + \epsilon\left(F_{LP}^D - 2F_5^D\right)\right] + \frac{\sin\theta}{2}\sqrt{2\epsilon(1+\epsilon)}\,F_5^D}{2\left((\epsilon-1)F_L^D + F_2^D\right)}$$

ϵ : *polarization parameter*

θ : *lepton scattering angle*

A good model for J Dirac structure

ratios of F_i^D

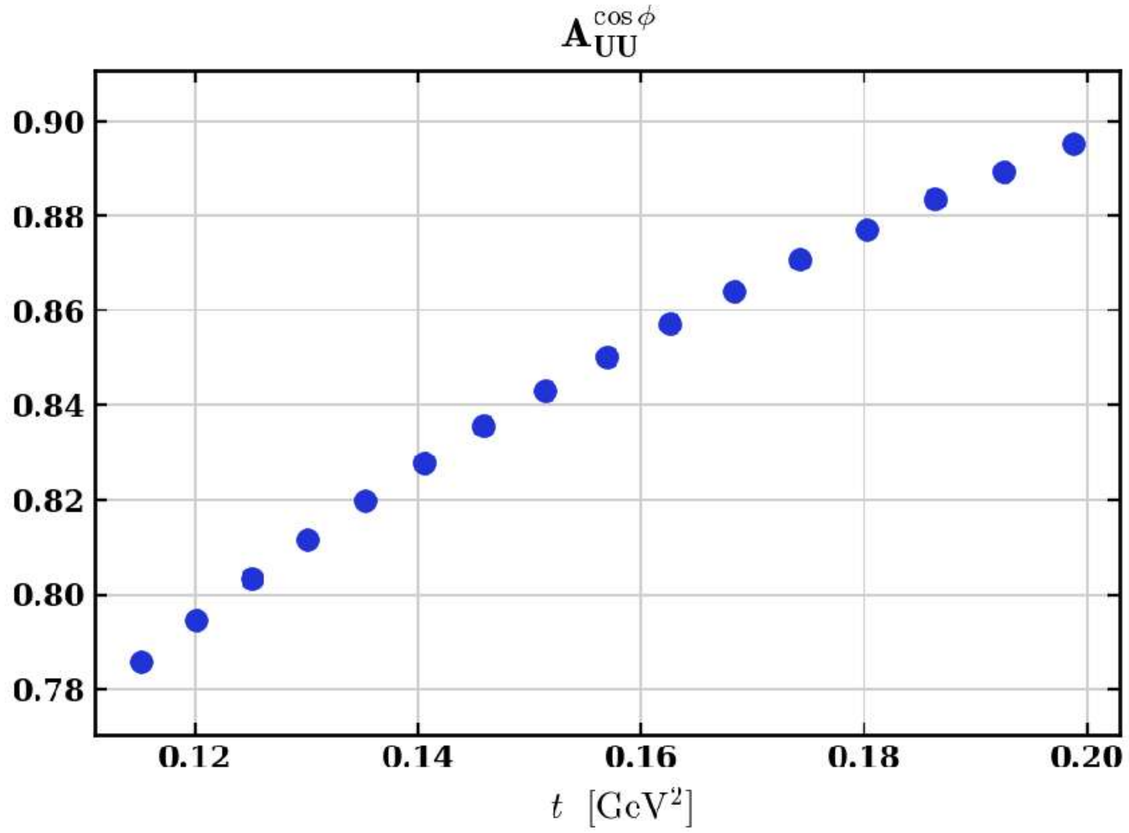
scalings for measured asymmetries

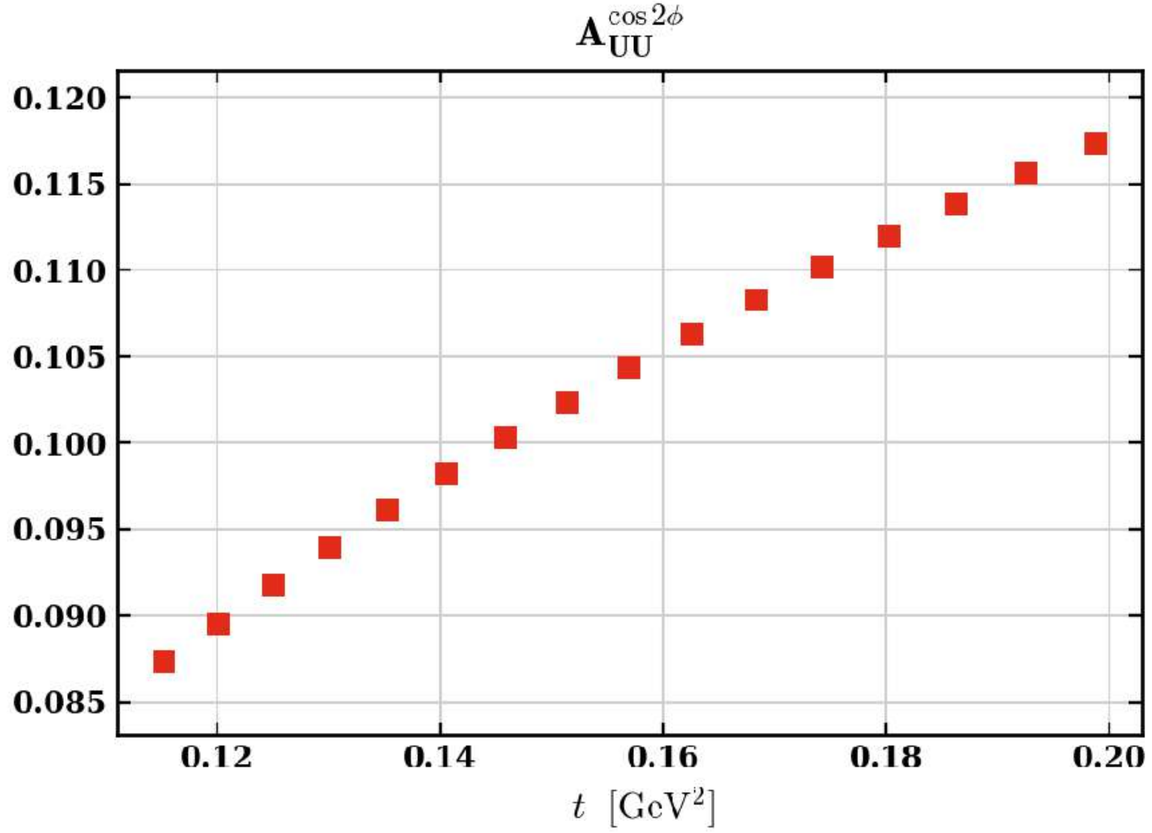
Unpolarized azimuthal modulations

Tell us whether unpolarized events prefer one direction in the azimuth.

Pion model predicts: $A_{UU}^{\cos\phi} = A_{UU}^{\cos 2\phi} = 0$

Our prediction:

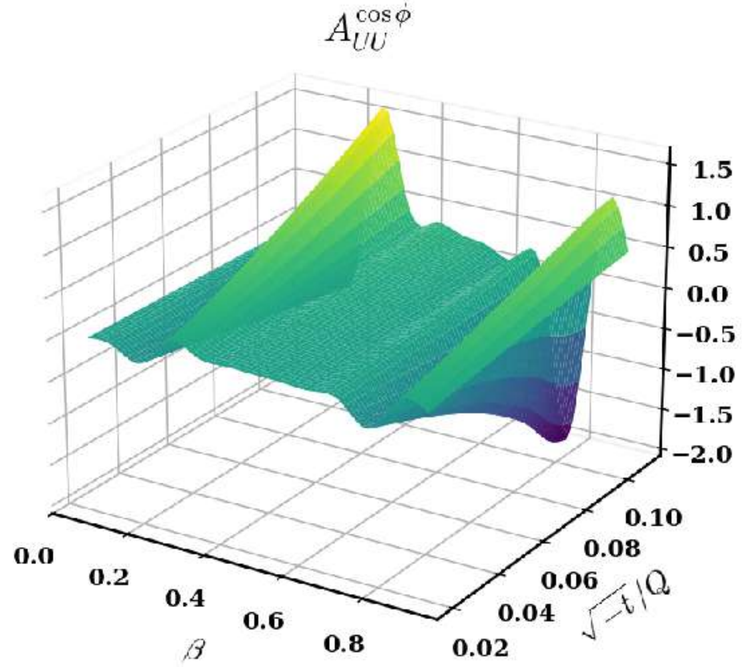


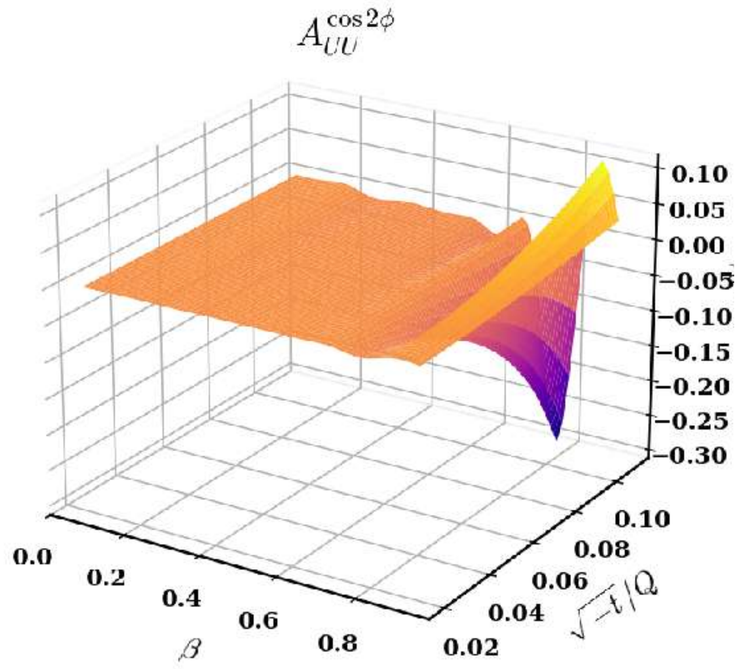


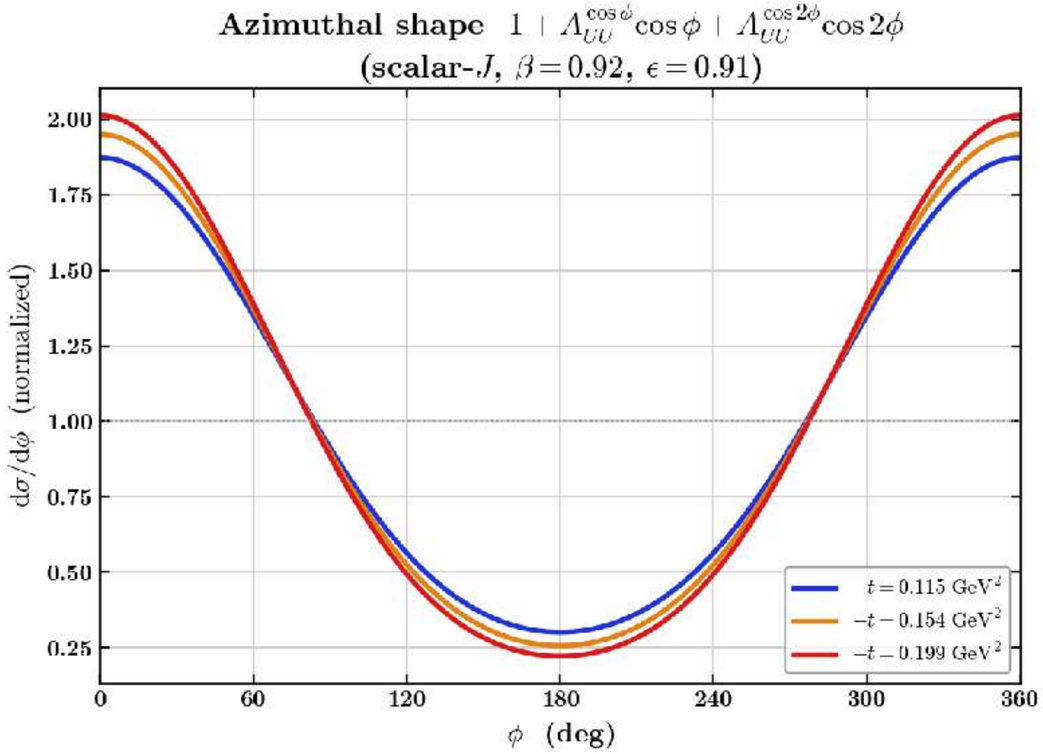
Clearly different results and physics from possible quark spin interferences!

Unpolarized azimuthal modulations

Tell us whether unpolarized events prefer one direction in the azimuth.





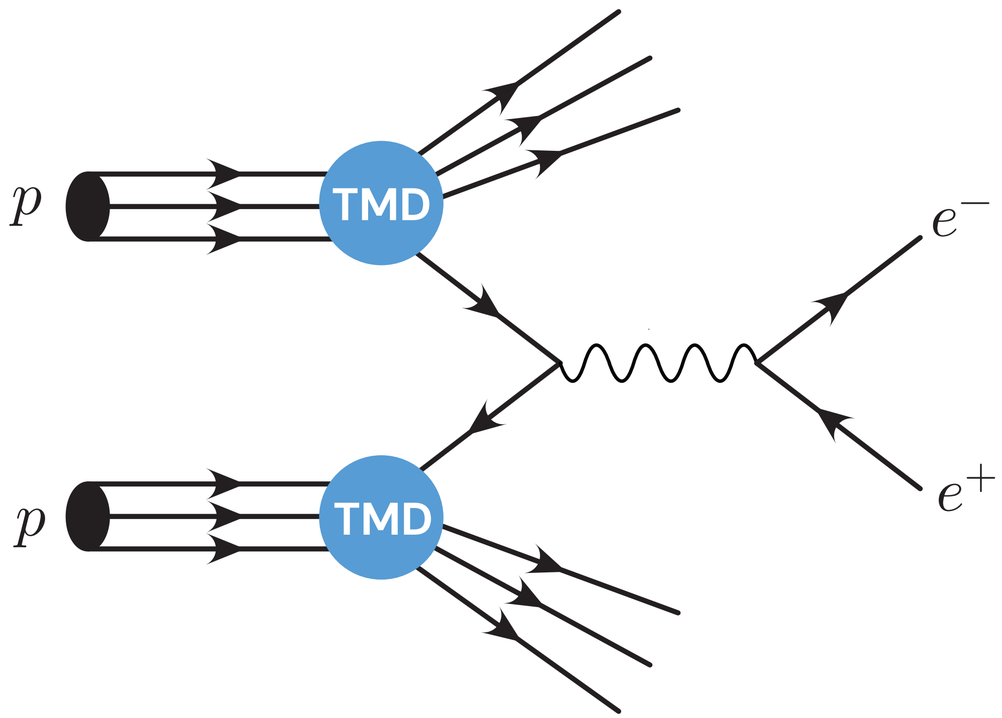


Although not yet measured for diffractive forward neutrons!

More things to see from EIC!

Universality

Drell-Yan

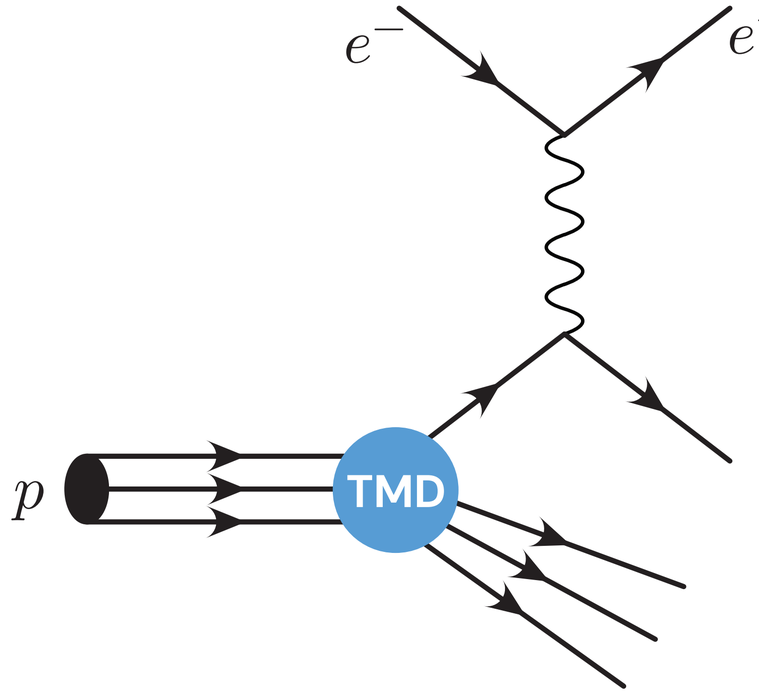


$$d\sigma_{\mathrm{DY}} \sim H \otimes f_{q/p_1}^{\mathrm{TMD}} \otimes f_{\bar{q}/p_2}^{\mathrm{TMD}}$$

← same → TMD/PDF input

universal non-perturbative input

SIDIS



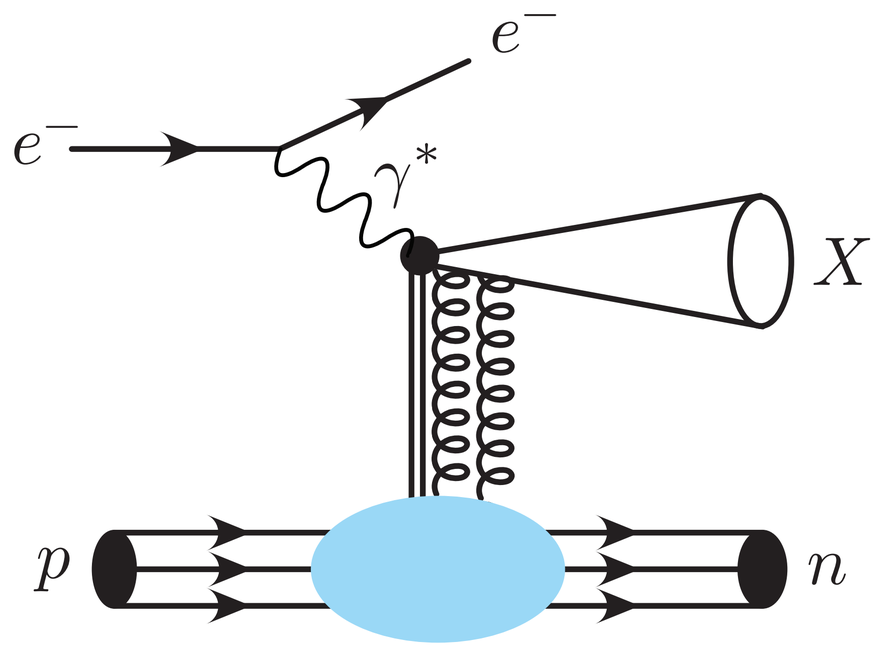
$$d\sigma_{\mathrm{SIDIS}} \sim H \otimes f_{q/p_1}^{\mathrm{TMD}} \otimes d^{\mathrm{TMDFF}}$$

Same TMD/PDF functions can be reused across processes

Universality

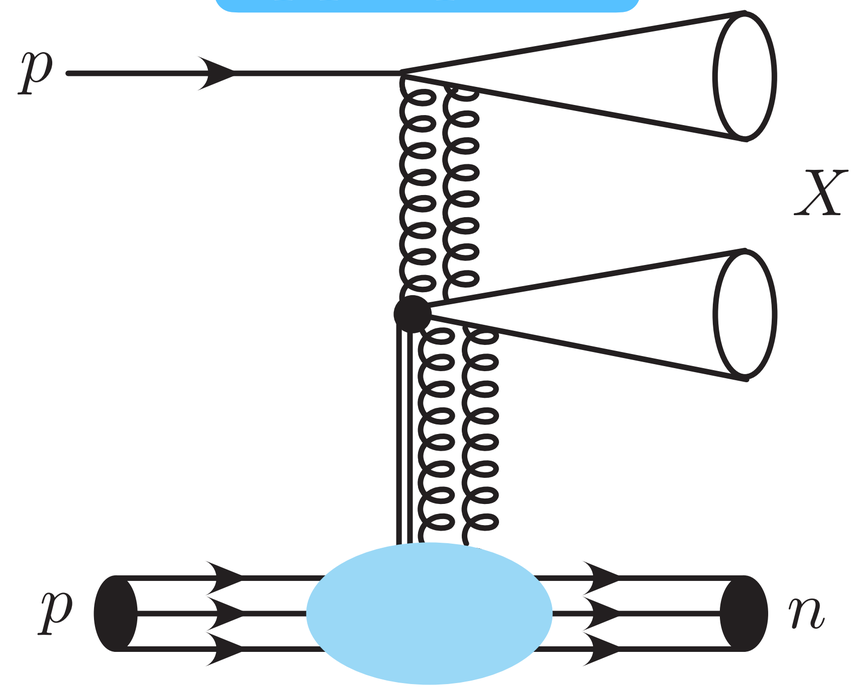
Is Sullivan universal to anything?

ep → eXn



$$d\sigma(ep \to eXn) \sim L \otimes (J^2_{p\to n} \otimes S)$$

pp→pXn



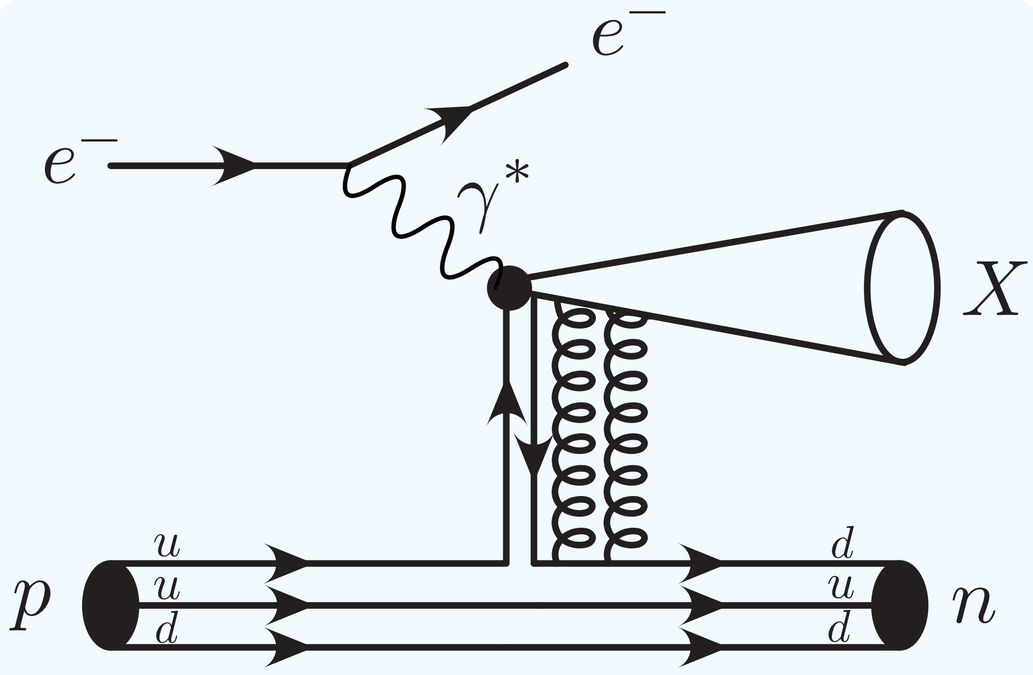
$$d\sigma(pp \to pXn) \sim B_{p\to p} \otimes (J^2_{p\to n} \otimes S)$$

Jet and its evolution is universal between EIC/RHIC

(Aretz, Lee, Schindler, Stewart 2607.07788)

Electroweak background

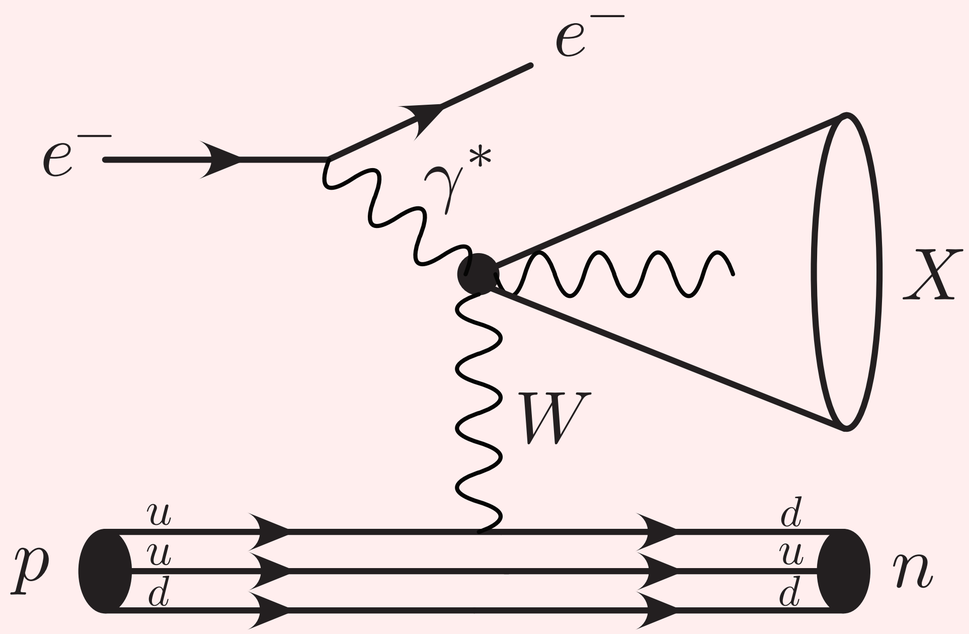
Dominant: Strong (Glauber exchange)



Glauber quark is suppressed by $\sqrt{\frac{-t}{s}}$

$$\sqrt{\frac{-t}{s}} \sim 10^{-3} - 10^{-2}$$

Suppressed: Electroweak



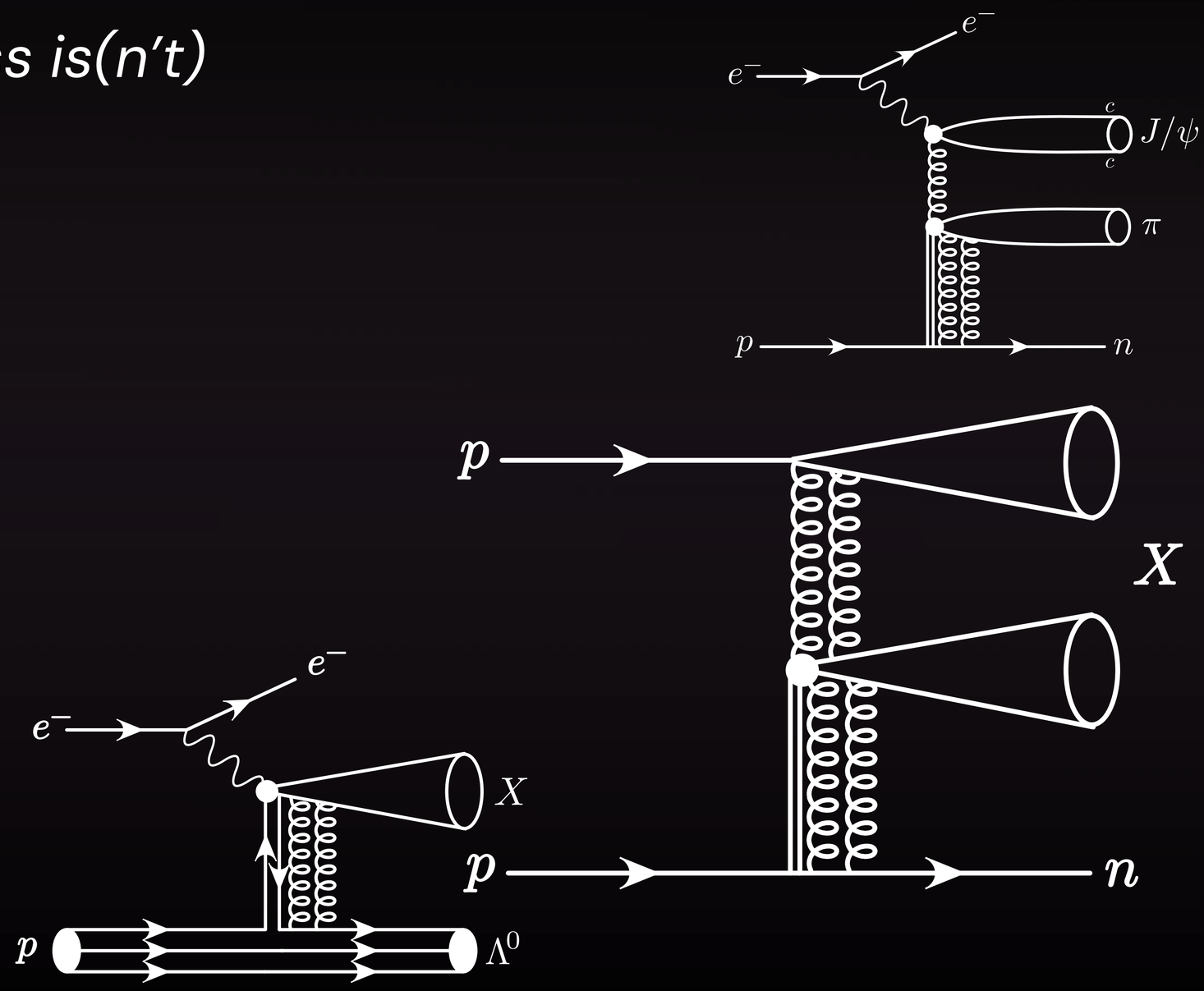
$p \to Wn$ transition is suppressed by G_F

$$G_F|t| \sim 10^{-5} - 10^{-4}$$

At EIC energies (28–140 GeV) strong dominates

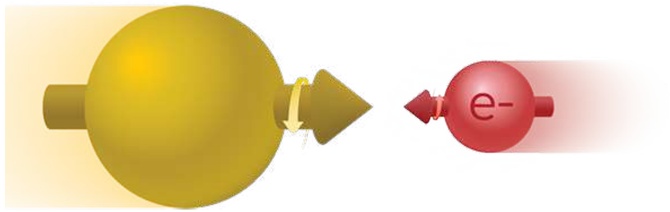
PART I. *What Sullivan process is(n't)*
PART II. *EFT for diffraction*
PART III. *Towards predictions*

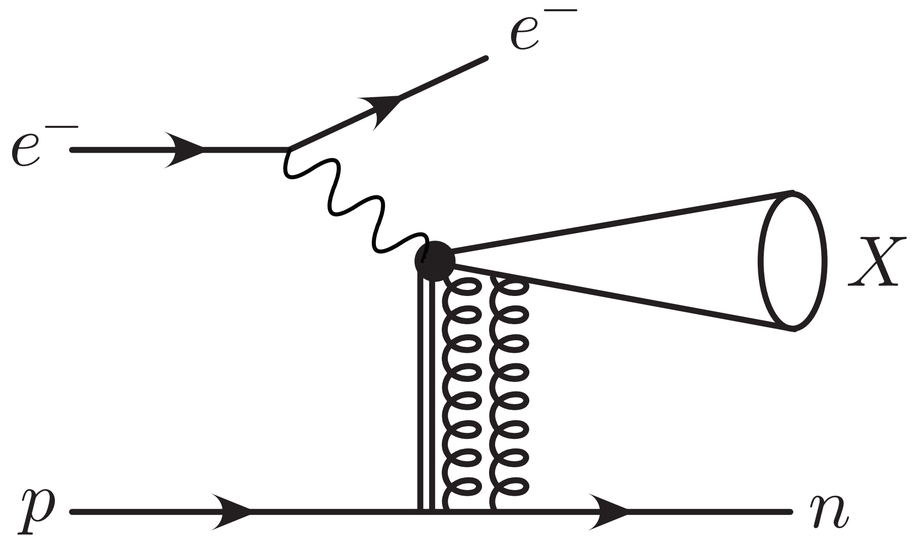
PART IV. *Extensions*



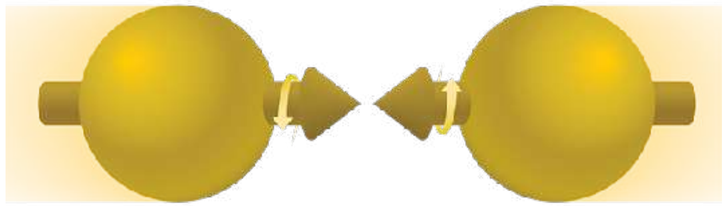
Natural extensions

$$ep \to eXn$$

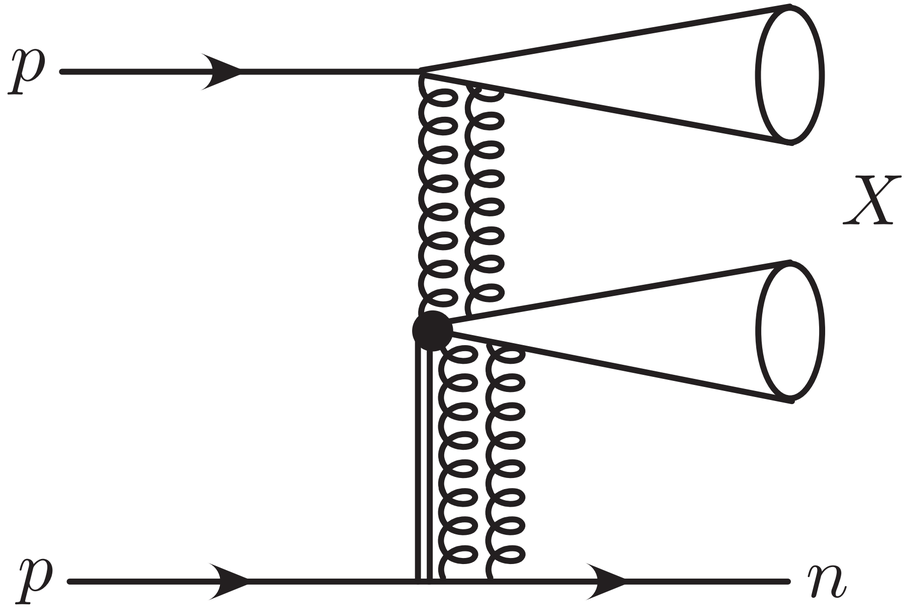




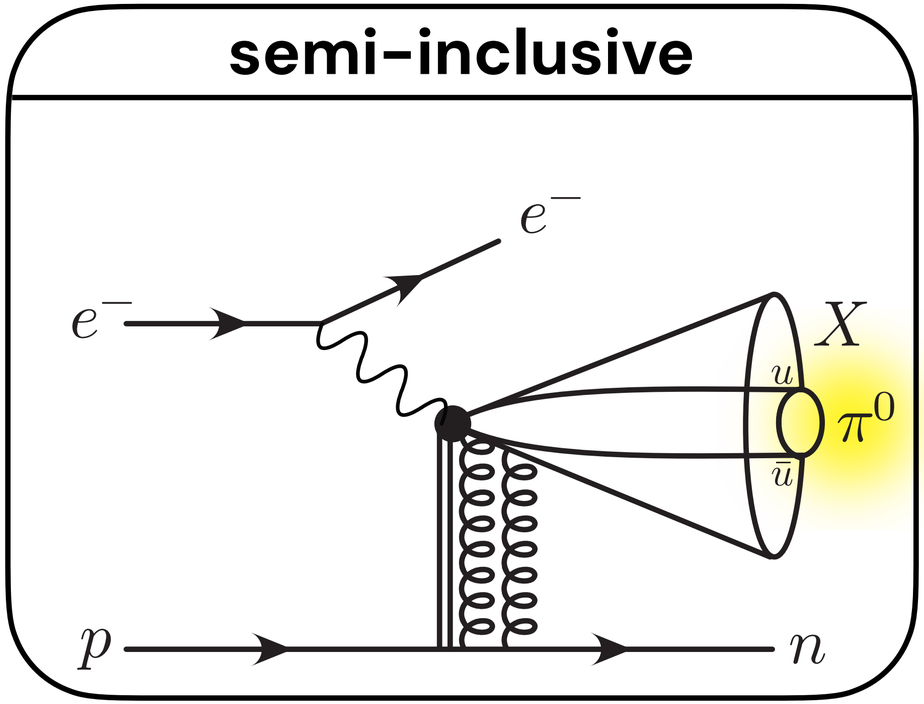
$$pp \to pXn$$

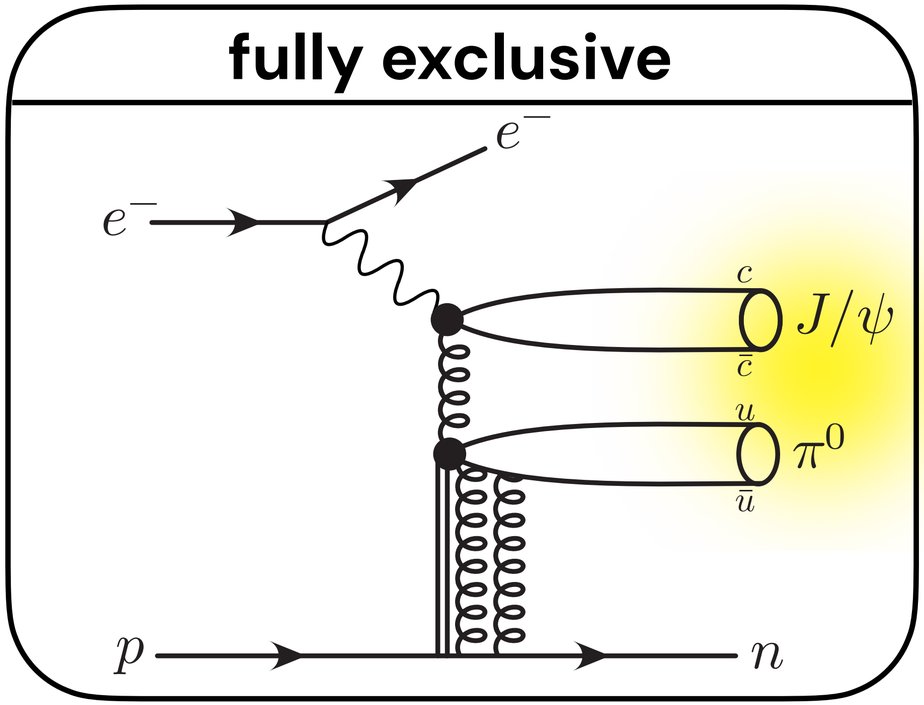


(RHIC already has relevant data)



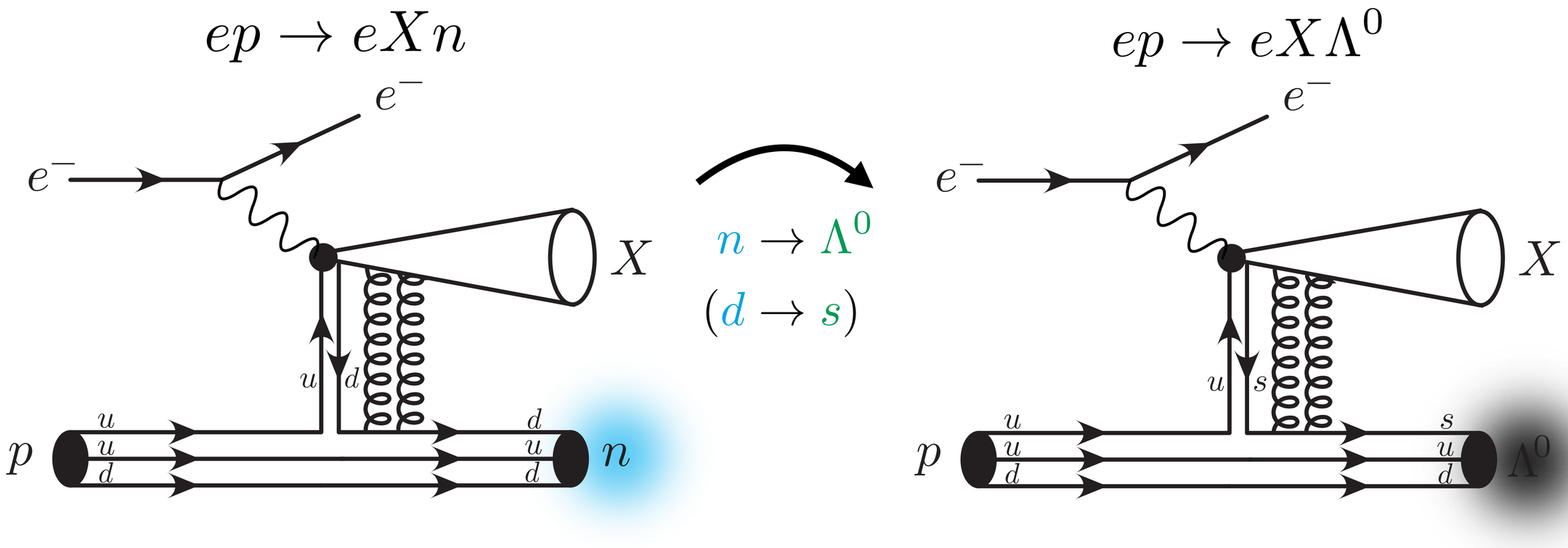
Additional particle tags





- Only **soft function** changes!
- A **heavy particle tag** (e.g. J/ψ) can introduce separated scales, so ***S* may refactorize**
- Tag leaves the general factorization/*J* structure intact, but factorization formula can develop **additional symmetries or cancellations**

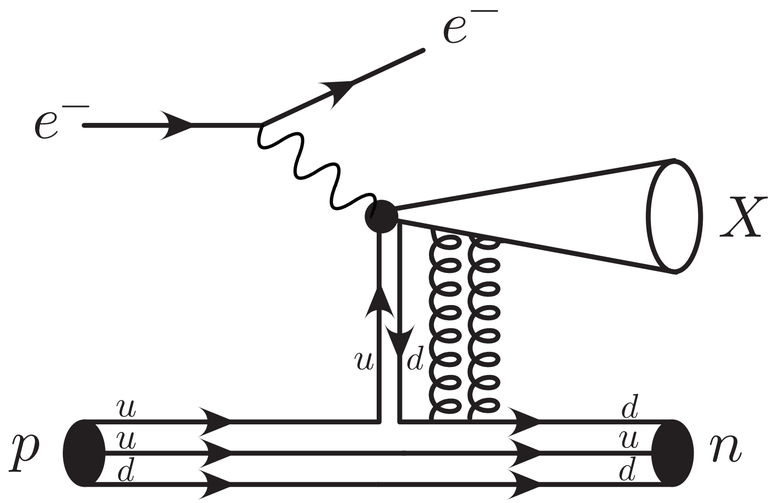
Other forward hadrons



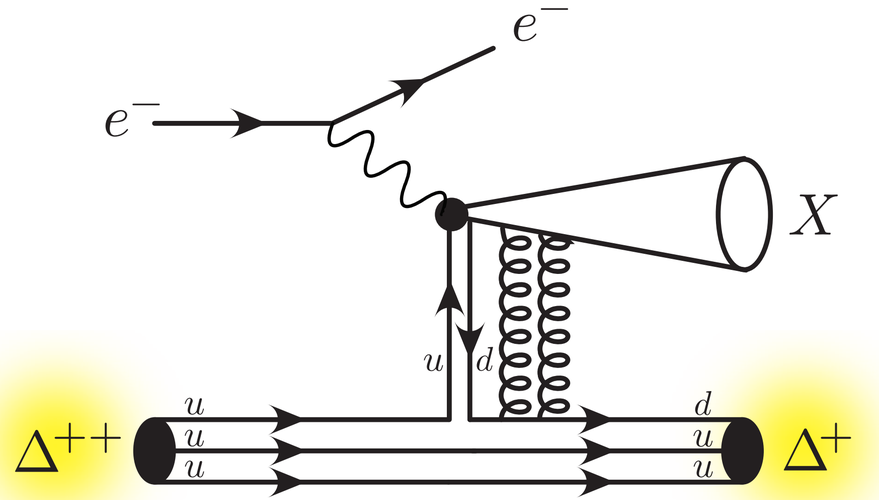
- **Jet function** and **convolution** change in a simple manner!

Other hadrons — "cousin" processes

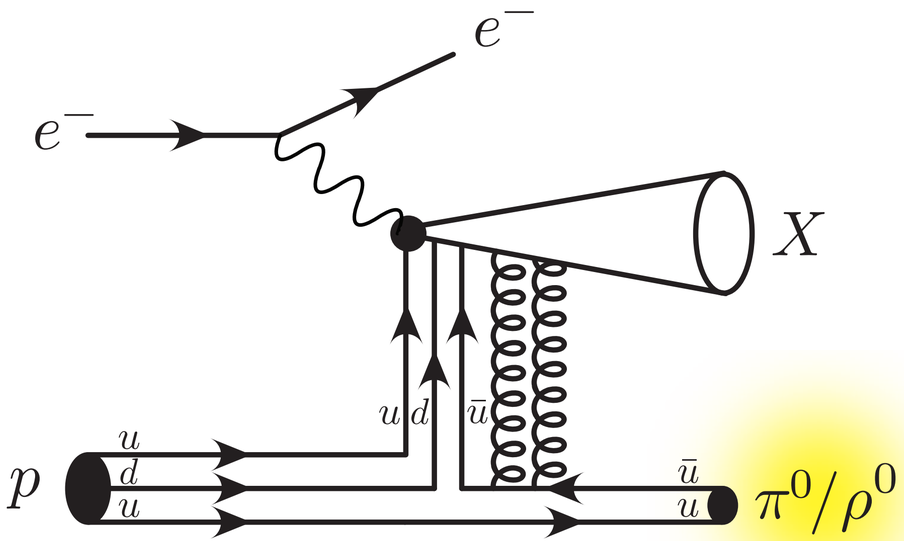
$ep \to eXn$



$e\Delta^{++} \to eX\Delta^+$



Same soft function and spin dependence

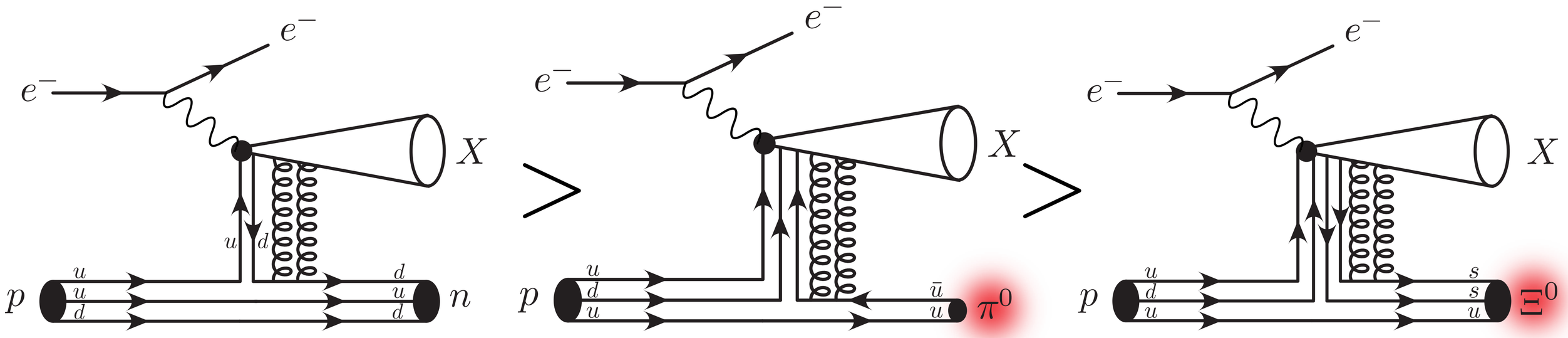


Same soft function operator structure but might be different in the spin projections

Other hadrons — hierarchies

$$\lambda = \frac{Q}{\sqrt{s}} \ll 1$$

In general, different numbers of exchanged Glauber quarks lead to hierarchies:



$$F_i^D(ep \to eXn) \sim \lambda^{-4}$$

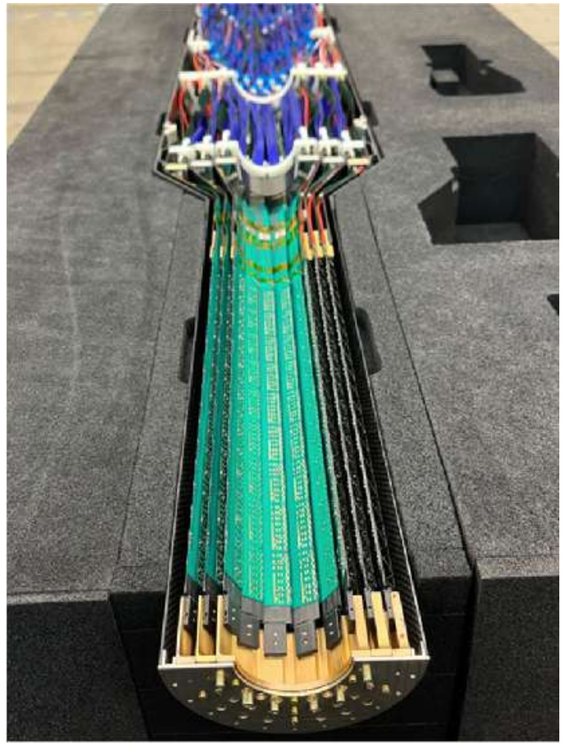
$$F_i^D(ep \to eX\pi^0) \sim \lambda^{-3}$$

$$F_i^D(ep \to eX\Xi^0) \sim \lambda^{-2}$$

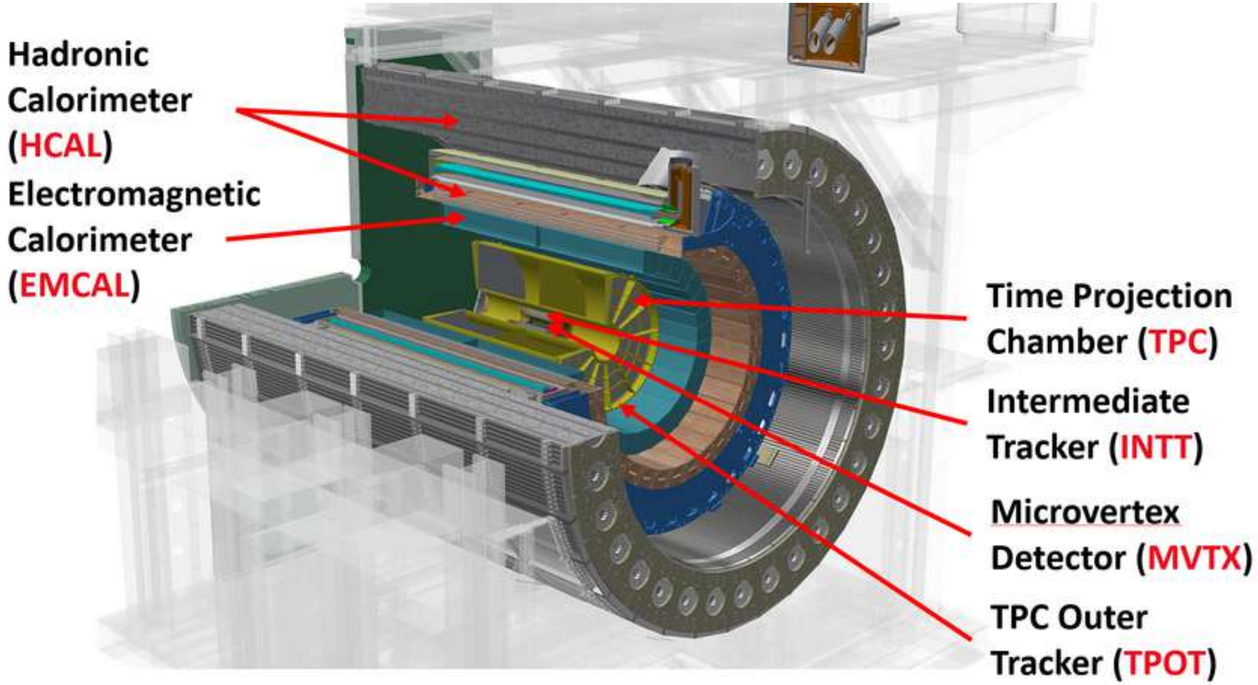
Other hadrons — experiments

Bottom line: lots of possible forward particles can be described!

- Current EIC ZDC **doesn't have particle ID**
- Have (new!) **sPHENIX silicon pixel vertex detectors** that can be repurposed for EIC
- Can study interesting **alternative forward processes in ZDC**



(Figure: BLN)



(Figure: J. Kvapil et al 2312.15104v2)

Theory – experiment symbiosis

Capitalize on the ZDC

Theory

Can estimate overall forward flux

→

Feasibility studies can help define realistic EIC observables

↗

↘

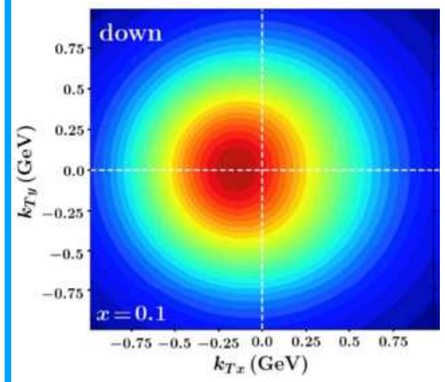
Overall forward-neutron measurements at the EIC look feasible

Can inform EIC detector design and science priorities

Experiment

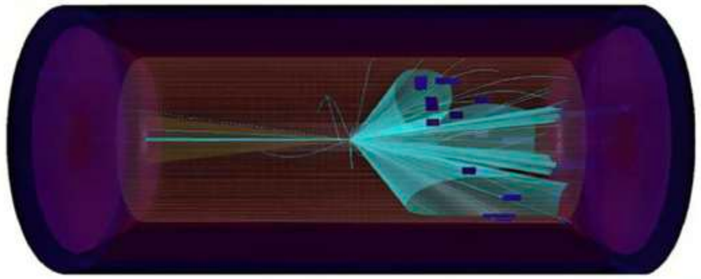
Broader impacts

hadron structure



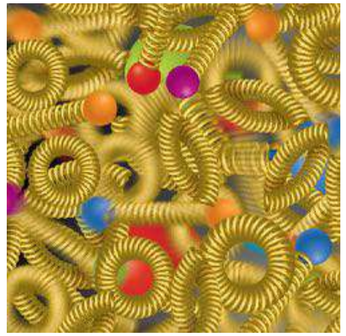
probe the proton-to-neutron transition

forward physics



inform EIC design and scientific goals

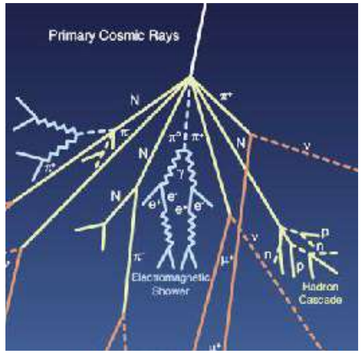
saturation



test nonlinear small-x dynamics

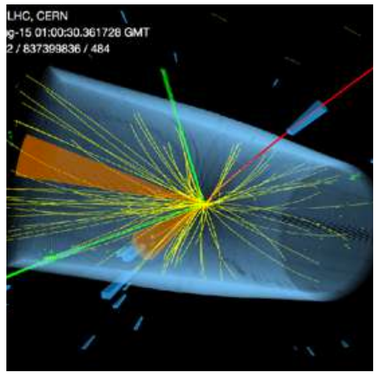
a major target for EIC!

cosmic rays



constrain hadronic MC modeling

new physics



gaps make our processes clean probes for new physics

Conclusion

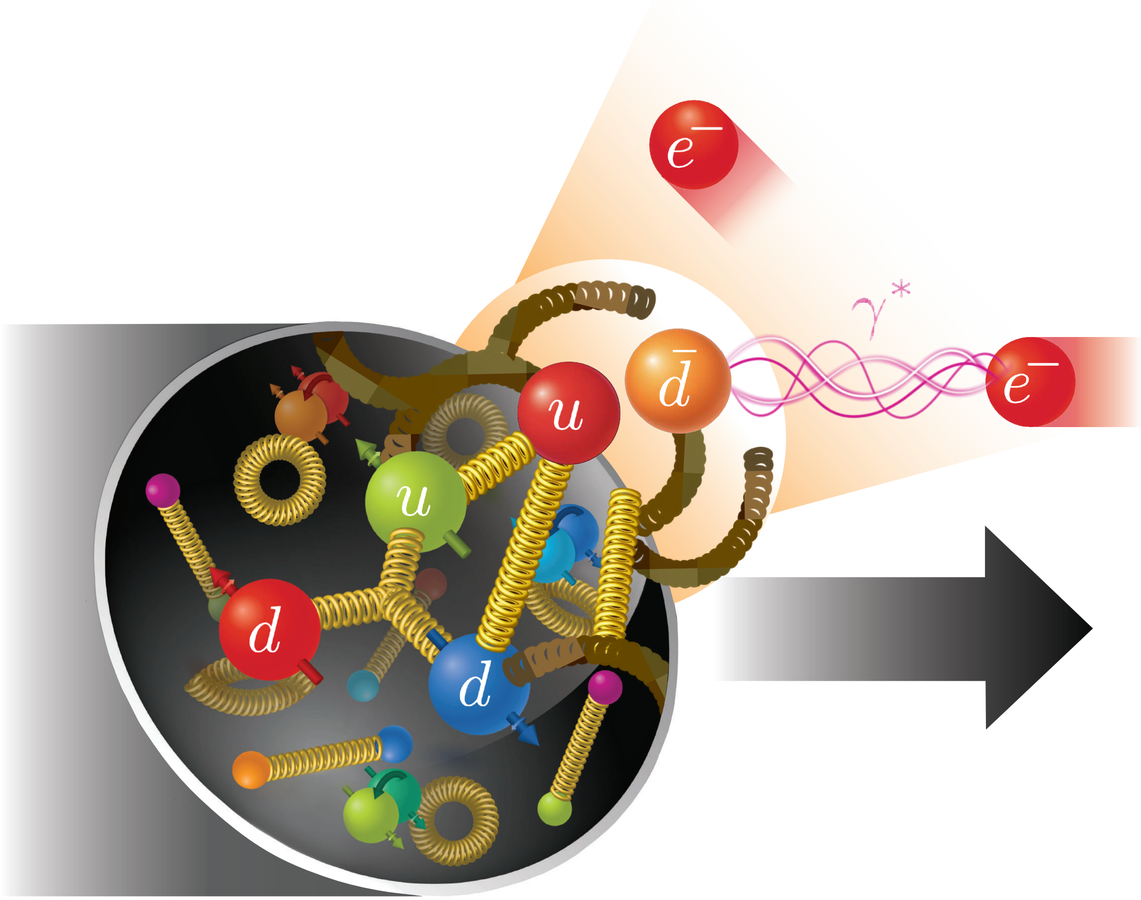
Sullivan process: not just pion exchange, but a diffractive mixed process

What we established

- the leading $p \to n$ transition enabled by Glauber quarks + Glauber gluons
- rigorous EFT framework for all flavor-enabled forward processes

What comes next

- model or constrain nonperturbative jet/ soft elements
- compare with existing HERA data
- develop predictions for EIC



Lots of interesting theoretical and experimental work before EIC's first light!

Acknowledgements

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Stella Schindler
Primary collaborator

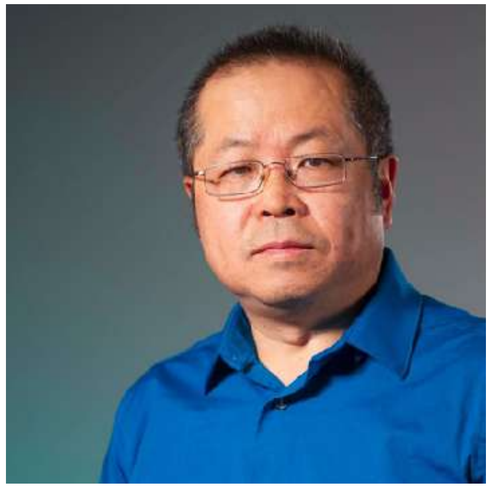


Christopher Lee



Duff Neill

LANL hosts



Ming Liu
LANL cohost
Future collaborator?

LANL'S LDRD Program

Backup

Lorentz invariants

2.1.1 Lorentz invariants

From four independent momentum measurements $\{k, k', p, p'\}$, we can form ten linearly independent Lorentz invariants. Of these, we let four vanish:

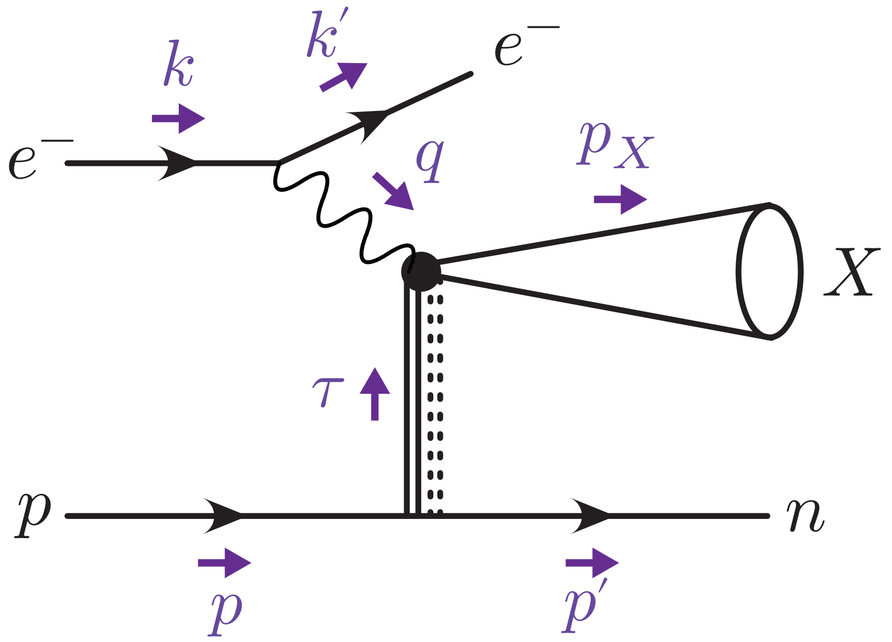
$$p^2 = m_p^2 \approx 0\,, \qquad (p')^2 = m_n^2 \approx 0\,, \qquad k^2 = (k')^2 = m_e^2 \approx 0\,, \tag{2.1}$$

dropping hadronic and leptonic mass corrections. A common choice for the six remaining independent invariants is

$$\begin{aligned} Q^2 &= -q^2\,, & t &= \tau^2\,, & \bar{x} &= \frac{k\cdot\tau}{k\cdot p}\,, \\ x &= \frac{Q^2}{2p\cdot q}\,, & y &= \frac{p\cdot q}{p\cdot k}\,, & \beta &= \frac{Q^2}{2q\cdot\tau}\,. \end{aligned} \tag{2.2}$$

It is conventional to define several auxiliary invariants:

$$\begin{aligned} W^2 &\equiv (p+q)^2 = \frac{1-x}{x}Q^2\,, & m_X^2 &\equiv p_X^2 = Q^2\frac{1-\beta}{\beta} + t\,, \\ z &\equiv \frac{p\cdot p'}{p\cdot q} = \frac{x}{Q^2}\,(m_Y^2 - t) \simeq \frac{x}{Q^2}\,(m_n^2 - t)\,, & s &= (p+k)^2 = \frac{Q^2}{xy} \\ \xi &\equiv x_{\mathbb{P}} = 1 - x_{\mathbb{L}} \equiv \frac{q\cdot\tau}{p\cdot q} = \frac{x}{\beta}\,. \end{aligned} \tag{2.3}$$



Structure functions

$$\frac{d^5\sigma}{dx\,dQ^2\,d\beta\,dt\,d\bar{x}} = \frac{\pi\alpha^2 x y^2}{4Q^6\beta^2 N_\sigma} L_{\mu\nu}(k,k')W^{\mu\nu}(p,q,p')\,,$$

$$W^{\mu\nu}(q,p,p') = \sum\!\!\!\!\!\!\int_X \delta^4(q+p-p'-p_X)\,\langle p|J^{\dagger\mu}(0)|nX\rangle\langle nX|J^{\nu}(0)|p\rangle\delta(p'^2-m_n^2),$$

$$\begin{aligned} L^{\mu\nu}(k,k') &= \langle k|J_e^{\dagger\mu}(0)|k'\rangle\langle k'|J_e^{\nu}(0)|k\rangle \\ &= 2\big[k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k'\,g^{\mu\nu} + \mathrm{i}\lambda_e\epsilon^{\mu\nu\rho\sigma}k_\rho k_\sigma\big]\,. \end{aligned}$$

$$W_D^{\mu\nu} = \sum_{n\in\{L,1,2,3,4,4A\ldots\}} F_n^D w_n^{\mu\nu}$$

$$w_L^{\mu\nu} = \frac{1}{2x}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right), \qquad w_2^{\mu\nu} = \frac{1}{2x}\left(U^\mu U^\nu - g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right),$$

$$w_3^{\mu\nu} = \frac{1}{2x}\left(2X^\mu X^\nu - U^\mu U^\nu + g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right), \qquad w_4^{\mu\nu} = \frac{1}{2x}\left(U^\mu X^\nu + X^\mu U^\nu\right),$$

$$w_{4A}^{\mu\nu} = \frac{i}{2x}\left(U^\mu X^\nu - X^\mu U^\nu\right), \qquad w_{LP}^{\mu\nu} = \tilde{S}_T\, w_L^{\mu\nu},$$

$$w_{2P}^{\mu\nu} = \tilde{S}_T\, w_2^{\mu\nu}, \qquad w_{3P}^{\mu\nu} = \tilde{S}_T\, w_3^{\mu\nu},$$

$$w_{4P}^{\mu\nu} = \tilde{S}_T\, w_4^{\mu\nu}, \qquad w_{4AP}^{\mu\nu} = \tilde{S}_T\, w_{4A}^{\mu\nu},$$

$$w_5^{\mu\nu} = \frac{1}{2x}\left(U^\mu \bar{U}^\nu + \bar{U}^\mu U^\nu\right), \qquad w_6^{\mu\nu} = \frac{1}{2x}\left(U^\mu \bar{X}^\nu + \bar{X}^\mu U^\nu\right),$$

$$w_7^{\mu\nu} = \frac{1}{2x}\left(X^\mu \bar{U}^\nu + \bar{U}^\mu X^\nu\right), \qquad w_8^{\mu\nu} = \frac{1}{2x}\left(X^\mu \bar{X}^\nu + \bar{X}^\mu X^\nu\right),$$

$$w_{5A}^{\mu\nu} = \frac{i}{2x}\left(U^\mu \bar{U}^\nu - \bar{U}^\mu U^\nu\right), \qquad w_{6A}^{\mu\nu} = \frac{i}{2x}\left(U^\mu \bar{X}^\nu - \bar{X}^\mu U^\nu\right),$$

$$w_{7A}^{\mu\nu} = \frac{i}{2x}\left(X^\mu \bar{U}^\nu - \bar{U}^\mu X^\nu\right), \qquad w_{8A}^{\mu\nu} = \frac{i}{2x}\left(X^\mu \bar{X}^\nu - \bar{X}^\mu X^\nu\right). \qquad (2.3)$$

Channel	Beam config.	# SFs	Characteristic modulations	Structure functions	Asymmetry type
UU	e unpol., p unpol.	4	$1, \cos\phi, \cos 2\phi$	F_L^D, F_2^D, F_3^D, F_4^D	No spin asymmetry
UL	e unpol., p_L pol.	2	$\sin\phi, \sin 2\phi$	$F_{UL}^{\sin\phi}$, $F_{UL}^{\sin 2\phi}$	Target SSA
LU	e_L pol., p unpol.	1	$\sin\phi$	F_{4A}^D	Beam SSA
LL	e_L pol., p_L pol.	2	$1, \cos\phi$	F_{LL}, $F_{LL}^{\cos\phi}$	Double-spin asymmetry (DSA)
UT	e unpol., p_T pol.	6	$\sin(\phi-\phi_S)$, $\sin(\phi+\phi_S)$, $\sin(3\phi-\phi_S)$, $\sin\phi_S$, $\sin(2\phi-\phi_S)$	$F_{UT,T}^{\sin(\phi-\phi_S)}$, $F_{UT,L}^{\sin(\phi-\phi_S)}$, $F_{UT}^{\sin(\phi+\phi_S)}$, $F_{UT}^{\sin(3\phi-\phi_S)}$, $F_{UT}^{\sin\phi_S}$, $F_{UT}^{\sin(2\phi-\phi_S)}$	Target SSA
LT	e_L pol., p_T pol.	3	$\cos(\phi-\phi_S)$, $\cos\phi_S$, $\cos(2\phi-\phi_S)$	$F_{LT}^{\cos(\phi-\phi_S)}$, $F_{LT}^{\cos\phi_S}$, $F_{LT}^{\cos(2\phi-\phi_S)}$	Double-spin asymmetry

$UU \leftrightarrow \{L,2,3,4\}$,
$LU \leftrightarrow 4A$,
$UT \leftrightarrow \{LP,2P,3P,4P\}$, i.e. transverse-spin copies of $\{L,2,3,4\}$,
$LT \leftrightarrow 4AP$,

while the remaining full-basis polarized structures live in

$$\{5,6,7,8,5A,6A,7A,8A\} \in UL, LL, UT, TT,$$

and are power suppressed in the leading diffraction factorization.

Structure functions

$$\frac{d^5\sigma(\lambda_\ell, S_L, S_T, \phi_S)}{dx\, dQ^2\, d\beta\, dt\, d\bar{x}} = \frac{\pi\alpha_{\rm em}^2}{2Q^4\beta^2 x\sqrt{(1-y)\left(z^2 - \frac{z}{\beta} - \frac{t}{Q^2}\right) - \frac{1}{4}\left(\frac{\bar{x}}{x} - \frac{1}{\beta} + (2-y)z\right)^2}}$$

$$\times \Bigg\{ -\frac{y^2}{2}F_L^D + \left(1 - y + \frac{y^2}{2}\right)F_2^D + (1-y)\cos(2\phi)\, F_3^D$$

$$- (2-y)\sqrt{1-y}\cos\phi\, F_4^D$$

$$+ \lambda_\ell\, y\sqrt{1-y}\sin\phi\, F_{4A}^D$$

$$+ S_L\Big[-(2-y)\sqrt{1-y}\sin\phi\, F_5^D + (1-y)\sin(2\phi)\, F_7^D\Big]$$

$$+ \lambda_\ell S_L\left[\frac{y(2-y)}{2}\, F_{7A}^D - y\sqrt{1-y}\cos\phi\, F_{5A}^D\right]$$

$$+ S_T\Bigg\{ \cos(\phi_J - \phi_S)\Big[(2-y)\sqrt{1-y}\sin\phi\, F_6^D - (1-y)\sin(2\phi)\, F_8^D\Big]$$

$$+ \sin(\phi - \phi_S)\Bigg[\left(1 - y + \frac{y^2}{2}\right)F_{2P}^D - \frac{y^2}{2}F_{LP}^D - 2(1-y)F_5^D$$

$$+ \left(1 - y + \frac{y^2}{2}\right)F_8^D$$

$$- (2-y)\sqrt{1-y}\cos\phi\left(F_{4P}^D + F_6^D - F_7^D\right)$$

$$+ (1-y)\cos(2\phi)\left(F_{3P}^D + F_8^D\right)\Bigg]\Bigg\}$$

$$+ \lambda_\ell S_T\Bigg\{ -\frac{y(2-y)}{2}\cos(\phi - \phi_S)\, F_{8A}^D$$

$$+ \frac{y\sqrt{1-y}}{2}\cos\phi_S\left(F_{4AP}^D + 2F_{6A}^D + F_{7A}^D\right)$$

$$+ \frac{y\sqrt{1-y}}{2}\cos(2\phi - \phi_S)\left(-F_{4AP}^D + F_{7A}^D\right)\Bigg\}\Bigg\}.$$

SCET modes in diffraction

In Breit frame

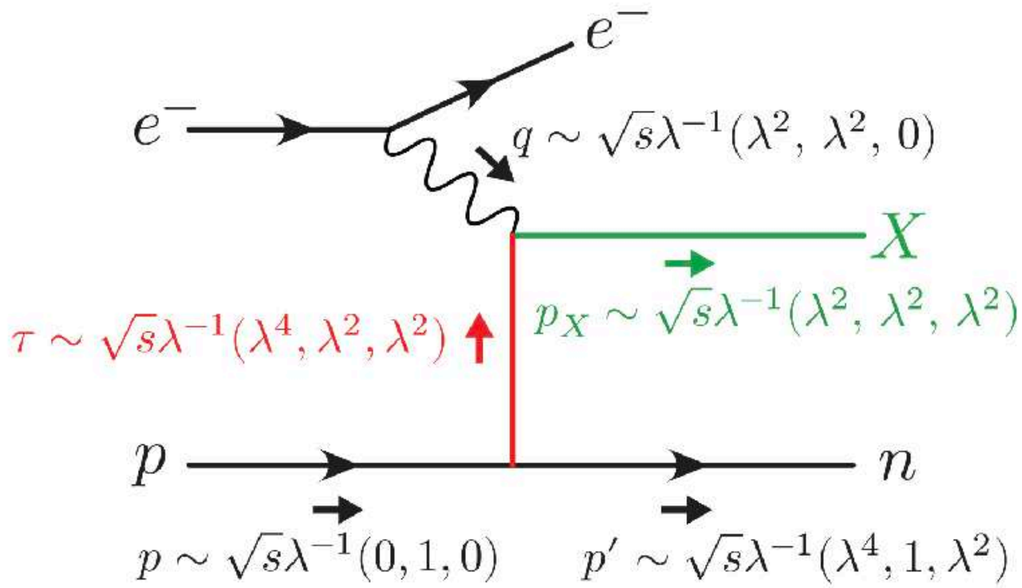
$$q^\mu = (0,0,0,-Q) = Q(1,-1,0_\perp)$$

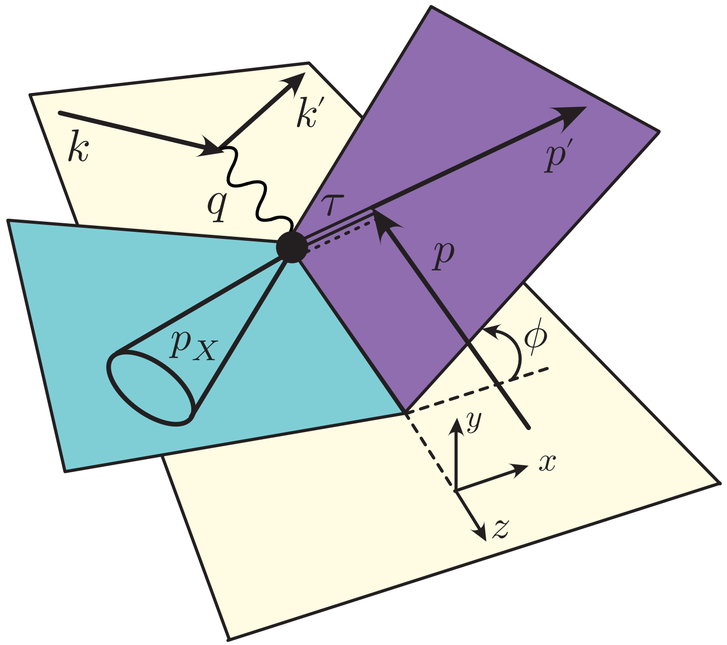
$$p^\mu = \frac{m_p^2}{n^-}\frac{\bar{n}^\mu}{2} + p^- \frac{n^\mu}{2} \approx Q\Big(0, \frac{1}{x}, 0_\perp\Big)$$

$$\tau = Q\Big(-z, \frac{1}{\beta} - z, \sqrt{-t/Q^2 - z(1/\beta - z)}\Big)\,,$$

$$p' = Q\Big(z, \frac{1}{x} - \frac{1}{\beta} + z, \sqrt{-t/Q^2 - z(1/\beta - z)}\Big)\,,$$

$$p_X = Q\Big(1-z, \frac{1-\beta}{\beta} - z, \sqrt{-t/Q^2 - z(1/\beta - z)}\Big)$$





$$p \sim \sqrt{s}(0, \lambda^{-1}, 0)\,,$$
$$q \sim \sqrt{s}(\lambda, \lambda, 0)\,,$$
$$p_X \sim \sqrt{s}(\lambda, \lambda/\beta, \lambda\lambda_t)\,,$$
$$p' \sim \sqrt{s}(\lambda^3\lambda_t^2, \lambda^{-1}, \lambda\lambda_t)$$
$$\tau \sim \sqrt{s}(\lambda^3\lambda_t^2, \lambda/\beta, \lambda\lambda_t)$$

$$\lambda = \frac{Q}{\sqrt{s}}, \qquad \lambda_t = \frac{\sqrt{-t}}{Q}. \qquad \lambda_\Lambda = \frac{\Lambda_{\rm QCD}}{\sqrt{-t}}$$

Glauber quarks

$$\mathcal{L}_{\rm QCD} \quad \xrightarrow{\lambda \ll 1} \quad \mathcal{L}^{(0)}_{\rm SCET} = \mathcal{L}_{\rm hard} + \mathcal{L}_{\rm collinear} + \mathcal{L}_{\rm soft} + \mathcal{L}_{\rm ultrasoft} + \mathcal{L}_{\rm Glauber}$$

$$\mathcal{L}_G^{(1/2)} \supset e^{-ix\cdot\mathcal{P}} \sum_n \bar{\mathcal{O}}_n \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_s^n + \bar{\mathcal{O}}_s^n \frac{1}{\not{\mathcal{P}}_\perp} \mathcal{O}_n \,. \tag{2.35}$$

The superscript $\frac{1}{2}$ indicates that this Lagrangian is at power $\lambda^{1/2}$ relative to leading power. Here we have the operators

$$\mathcal{O}_n = \not{\mathcal{B}}_{\perp n} \chi_n \,, \qquad \mathcal{O}_s^n = -4\pi\alpha_s \not{\mathcal{B}}^n_{\perp s} \psi^n_s \,, \qquad \bar{\mathcal{O}}_s^n = -4\pi\alpha_s \bar{\psi}^n_s \not{\mathcal{B}}^n_{\perp s} \,. \tag{2.36}$$

	Fundamental	Definition	Adjoint	Definition
Collinear fermion	χ_n	$W_n^\dagger \xi_n$	–	–
Soft fermion	ψ_s^n	$S_n^\dagger \psi_s$	–	–
Collinear gluon	$\mathcal{B}^{\mu A}_{n\perp}$	$\frac{1}{g} W_n^\dagger i D^\mu_{n\perp} W_n$	$\mathcal{B}^{\mu}_{n\perp}$	$\mathcal{B}^{\mu A}_{n\perp} T^A$
Soft gluon	$\mathcal{B}^{n\mu A}_{s\perp}$	$\frac{1}{g} S_n^\dagger i D^\mu_{s\perp} S_n$	$\mathcal{B}^{n\mu}_{s\perp}$	$\mathcal{B}^{n\mu A}_{s\perp} T^A$

Glauber gluons

Glauber gluons. At leading power, the Glauber Lagrangian [75] is

$$\mathcal{L}_G^{(0)} = e^{-ix\cdot\mathcal{P}} \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_{ns\bar{n}}^{ij} + e^{-ix\cdot\mathcal{P}} \sum_{n} \sum_{i,j=q,g} \mathcal{O}_{ns}^{ij} \,, \tag{2.31}$$

where the two- and three-rapidity-sector Glauber operators are

$$\mathcal{O}_{ns}^{ij} = \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B} \qquad \mathcal{O}_{ns\bar{n}}^{ij} = \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} \,. \tag{2.32}$$

The building block definitions of these operators are:

$$\begin{aligned}
\mathcal{O}_n^{qB} &= \frac{1}{2} \bar{\chi}_n T^B \bar{n}\!\!\!/ \chi_n & \mathcal{O}_{\bar{n}}^{qB} &= \frac{1}{2} \bar{\chi}_{\bar{n}} T^B n\!\!\!/ \chi_{\bar{n}} \\
\mathcal{O}_n^{gB} &= \frac{i}{4} f^{BCD} \mathcal{B}_{n\perp\mu}^C \Big[\bar{n}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\Big] \mathcal{B}_{n\perp}^{D\mu} & \mathcal{O}_{\bar{n}}^{gB} &= \frac{i}{4} f^{BCD} \mathcal{B}_{\bar{n}\perp\mu}^C \Big[n\cdot(\mathcal{P}+\mathcal{P}^\dagger)\Big] \mathcal{B}_{\bar{n}\perp}^{D\mu} \\
\mathcal{O}_s^{q_n B} &= 4\pi\alpha_s (\bar{\psi}_s^n T^B \bar{n}\!\!\!/ \psi_s^n) & \mathcal{O}_s^{q_{\bar{n}} B} &= 4\pi\alpha_s (\bar{\psi}_s^{\bar{n}} T^B n\!\!\!/ \psi_s^{\bar{n}}) \\
\mathcal{O}_s^{g_n B} &= 2\pi i \alpha_s f^{BCD} \mathcal{B}_{s\perp\mu}^{nC} \Big[n\cdot(\mathcal{P}+\mathcal{P}^\dagger)\Big] \mathcal{B}_{s\perp}^{nD\mu} & \mathcal{O}_s^{g_{\bar{n}} B} &= 2\pi i \alpha_s f^{BCD} \mathcal{B}_{s\perp\mu}^{\bar{n}C} \Big[\bar{n}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\Big] \mathcal{B}_{s\perp}^{\bar{n}D\mu}
\end{aligned} \tag{2.33}$$

as well as

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big[\mathcal{P}_\perp^\mu S_n^T S_{\bar{n}} \mathcal{P}_{\perp\mu} - g\mathcal{P}_\perp^\mu \tilde{B}_{S\perp}^{n\mu} S_n^T S_{\bar{n}} - g S_n^T S_{\bar{n}} \tilde{B}_{s\perp}^{\bar{n}\mu} - g^2 \tilde{B}_{s\perp}^{n\mu} S_n^T S_{\bar{n}} \tilde{B}_{s\perp\mu}^{\bar{n}} - \frac{ig}{2} n_\mu \bar{n}_\nu S_n^T \tilde{G}_s^{\mu\nu} S_{\bar{n}} \Big]^{BC}$$

A useful shorthand for eq. (2.31), which makes the sum over i, j implicit, is

$$\mathcal{O}_n^B = \mathcal{O}_n^{qB} + \mathcal{O}_n^{gB} \,, \qquad \mathcal{O}_s^B = \mathcal{O}_s^{q_n B} + \mathcal{O}_s^{g_n B} \,. \tag{2.34}$$

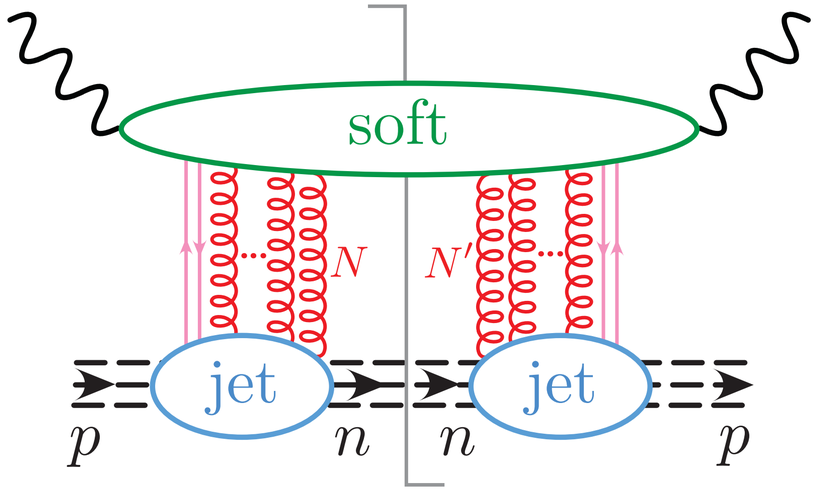
Note that Glauber gluons carry no power suppression in λ; i.e., each is of order $\sim \mathcal{O}(\lambda^0)$.

Factorization of the Sullivan process

$$F_i^D = S_{iNN'} \otimes_\perp (J_N \times J_{N'})$$

$$J_N^{R^N} = \frac{1}{\bar{n}\cdot p}\langle p|\Big\{\mathcal{O}_n(0,\tau_{u\perp})\bar{\mathcal{O}}_n(0,\tau_{\bar{d}\perp})\prod_{i=1}^{N-1}\mathcal{O}_n^{A_i}(0,\tau_{i\perp})\hat{\mathcal{O}}_n^{A_N}(0)\Big\}P_N^{R^N}|n\rangle$$

$$S_{iNN'}^{R^N R^{N'}} = \left(\frac{Q}{n\cdot q}\right)^2 \mathcal{P}'_{i\,\mu\nu}\int [d\tilde{y}][d\tilde{y}']đ^{\,d}z e^{iz\cdot q}\sum\!\!\!\!\!\!\int_{X_s}\langle 0|\bar{T}J_s^\mu(z)\Big\{\bar{\mathcal{O}}_n^s(\tilde{y},-\tau_{u\perp})\mathcal{O}_n^s(\tilde{y},-\tau_{\bar{d}\perp})\prod_{i=1}^{N}\mathcal{O}_s^{A_i}(\tilde{y},\tau_{i\perp})\Big\}P_N^{R_A^N}|X_s\rangle$$
$$\times\langle X_s|P_{N'}^{R_{A'}^{N'}}TJ_s^\nu(0)\Big\{\bar{\mathcal{O}}_n^s(\tilde{y}',-\tau'_{u\perp})\mathcal{O}_n^s(\tilde{y}',-\tau'_{\bar{d}\perp})\prod_{j=1}^{N'}\mathcal{O}_s^{A'_i}(\tilde{y}',-\tau'_{i\perp})\Big\}|0\rangle$$



$$\iint_{(N,\,N')}^{\perp\, u\bar{d}} \equiv \frac{(-i)^N i^{N'}}{N!\,N'!}\int \frac{đ^{\,d'}\tau_{u\perp}}{\not{t}_{u\perp}}\frac{đ^{\,d'}\tau_{\bar{d}\perp}}{\not{t}_{\bar{d}\perp}}\prod_{i=1}^{N}\frac{đ^{d'}\tau_{i\perp}}{\vec{\tau}_{i\perp}^{\,2}}\,\delta^{\,d'}\Big(\tau_{u\perp}+\tau_{\bar{d}\perp}+\sum_i\tau_{i\perp}+\tau_\perp\Big)$$
$$\times\int\frac{đ^{\,d'}\tau'_{u\perp}}{\not{t}'_{u\perp}}\frac{đ^{\,d'}\tau'_{\bar{d}\perp}}{\not{t}'_{\bar{d}\perp}}\prod_{j=1}^{N'}\frac{đ^{d'}\tau'_{j\perp}}{\vec{\tau}_{j\perp}^{\,\prime 2}}\,\delta^{\,d'}\Big(\tau'_{u\perp}+\tau'_{\bar{d}\perp}+\sum_j\tau'_{j\perp}-\tau_\perp\Big)$$

Factorization of the Sullivan process

$$F_i^D = \frac{Q^2\varsigma^2}{x} \sum_{N,N'=1}^{\infty} \sum_{\{R_X\}=1} \iint_{NN'}^{\perp u\bar{d}} S_{i\,NN'}^{R^N R^{N'}}\left(q^+\tau^-, \{\tau_{(i,u,\bar{d})\perp}, \tau'_{(i,u,\bar{d})\perp}\}, Q, t\right)$$
$$\times J_N^{R^N}\left(p^- p'^+, \{\tau_{u\perp}, \tau_{\bar{d}\perp}, \tau_{i\perp}\}, t\right)\; J_{N'}^{R^{N'}}\left(p^- p'^+, \{\tau'_{u\perp}, \tau'_{\bar{d}\perp}, \tau'_{j\perp}\}, t\right),$$

With the explicit spinor indices $(\alpha,\beta,\gamma,\sigma),(\alpha',\beta',\gamma',\sigma')$ the soft and jet functions are schematically

$$J^{\alpha\gamma} \propto \langle p|\{\!\{\mathcal{O}_n^\alpha \mathcal{O}_n^\gamma \prod \mathcal{O}_n^g\}\!\}|n\rangle\,,$$
$$S^{\beta\sigma,\beta'\sigma'} \propto \langle 0|\bar{T}J\{\!\{\bar{\mathcal{O}}_n^{s,\beta}\mathcal{O}_n^{s,\sigma}\prod \mathcal{O}_s^g\}\!\}P|X_s\rangle\langle X_s|PTJ\{\!\{\bar{\mathcal{O}}_n^{s,\beta'}\mathcal{O}_n^{s,\sigma'}\prod \mathcal{O}_s^g\}\!\}|0\rangle,$$
$$\iint^{\perp u\bar{d}} \propto \int \frac{\bar{d}^{\,d'}\tau_{u\perp}}{t_{u\perp}^{\alpha\beta}} \frac{\bar{d}^{\,d'}\tau_{\bar{d}\perp}}{t_{\bar{d}\perp}^{\gamma\sigma}} \cdots \int \frac{\bar{d}^{\,d'}\tau'_{u\perp}}{t'_{u\perp}{}^{\beta'\alpha'}} \frac{\bar{d}^{\,d'}\tau'_{\bar{d}\perp}}{t'_{\bar{d}\perp}{}^{\sigma'\gamma'}} \cdots .$$

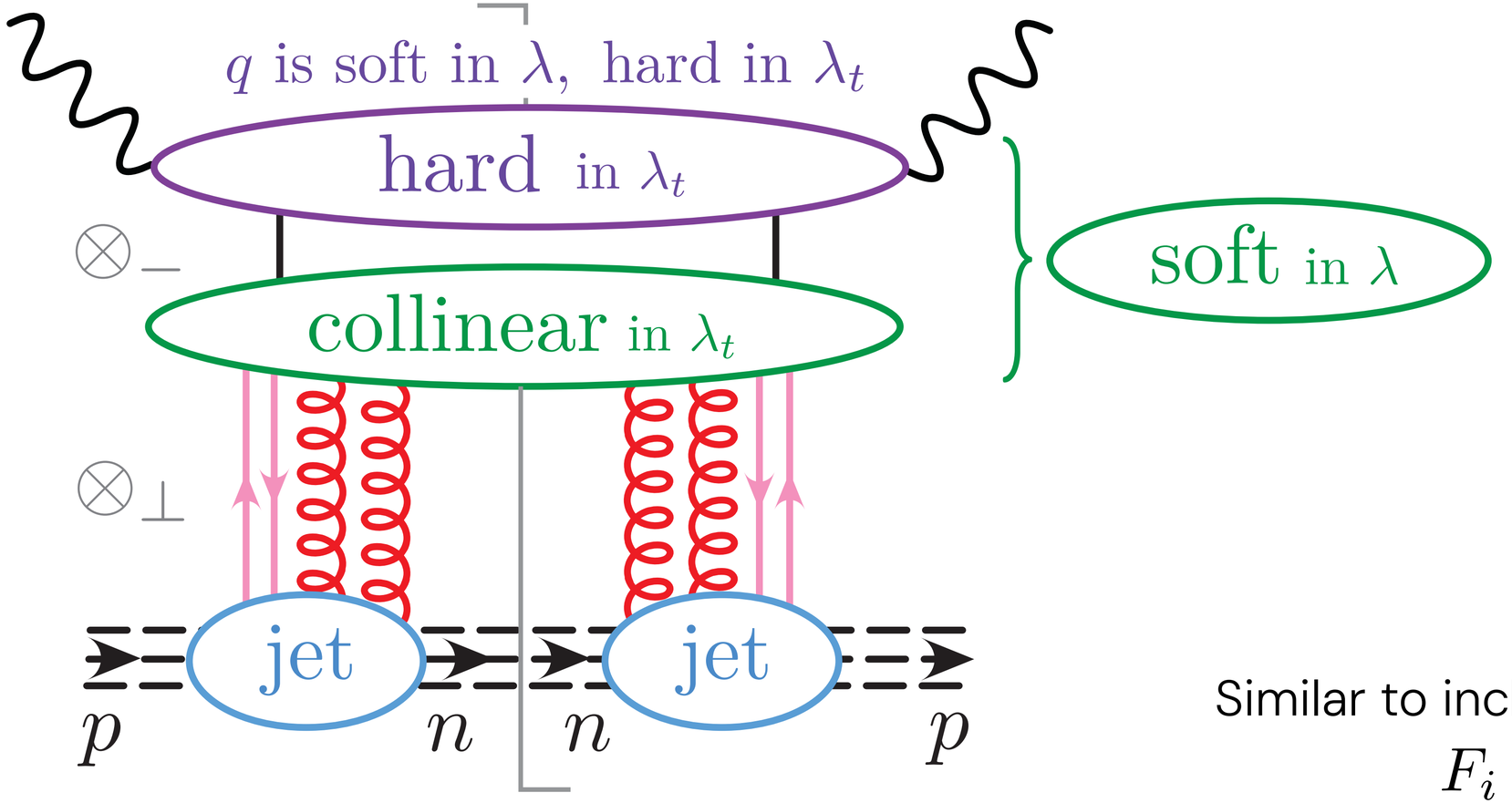
Hard-collinear refactorization

Kinematic regime

$$\lambda_t \equiv \frac{\sqrt{-t}}{Q} \ll 1$$

Refactorized form

$$F_i^D \sim S_i \otimes_\perp J^2 \xrightarrow{|t|^2 \ll Q^2} [H_i^{\rm DIS} \otimes_- S_c] \otimes_\perp J^2$$



$H_i^{\rm DIS}$	Hard DIS coefficient
S_c	Soft-collinear function soft in λ collinear in λ_t
J^2	Non-perturbative jet function

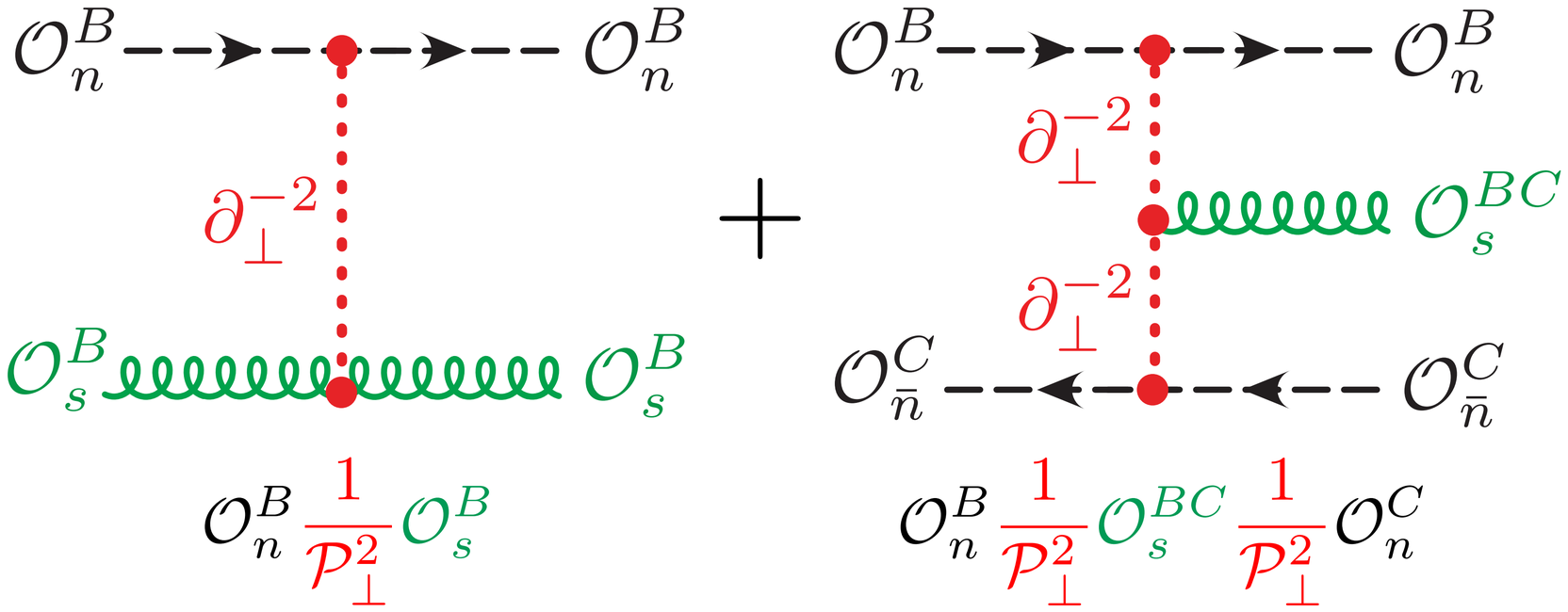
Similar to inclusive DIS factorization

$$F_i \sim H_i^{\rm DIS} \otimes f_i$$

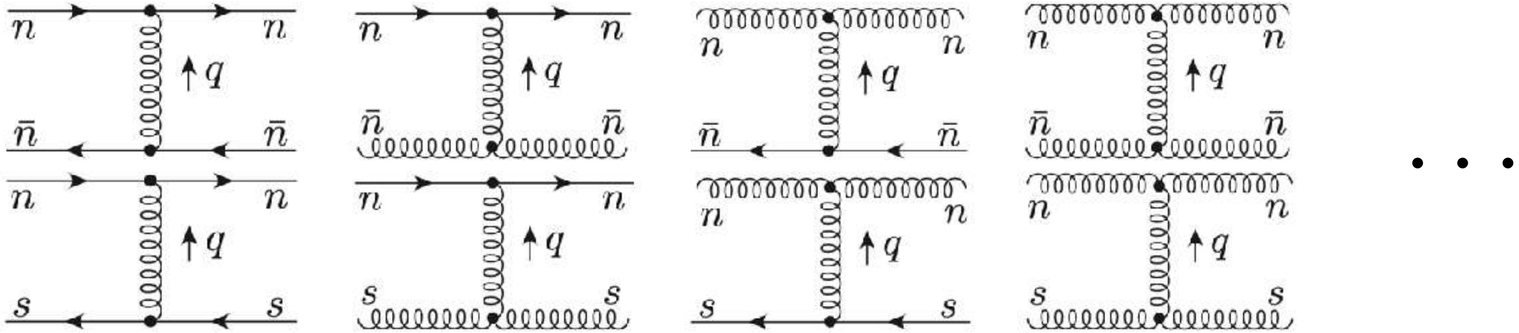
Glauber gluons

Leading power ‘connections’ between sectors

$$\mathcal{L}^{(0)}_{\text{Glauber}} = \mathcal{O}(\lambda^0)$$



derived from tree-level matching to QCD diagrams:

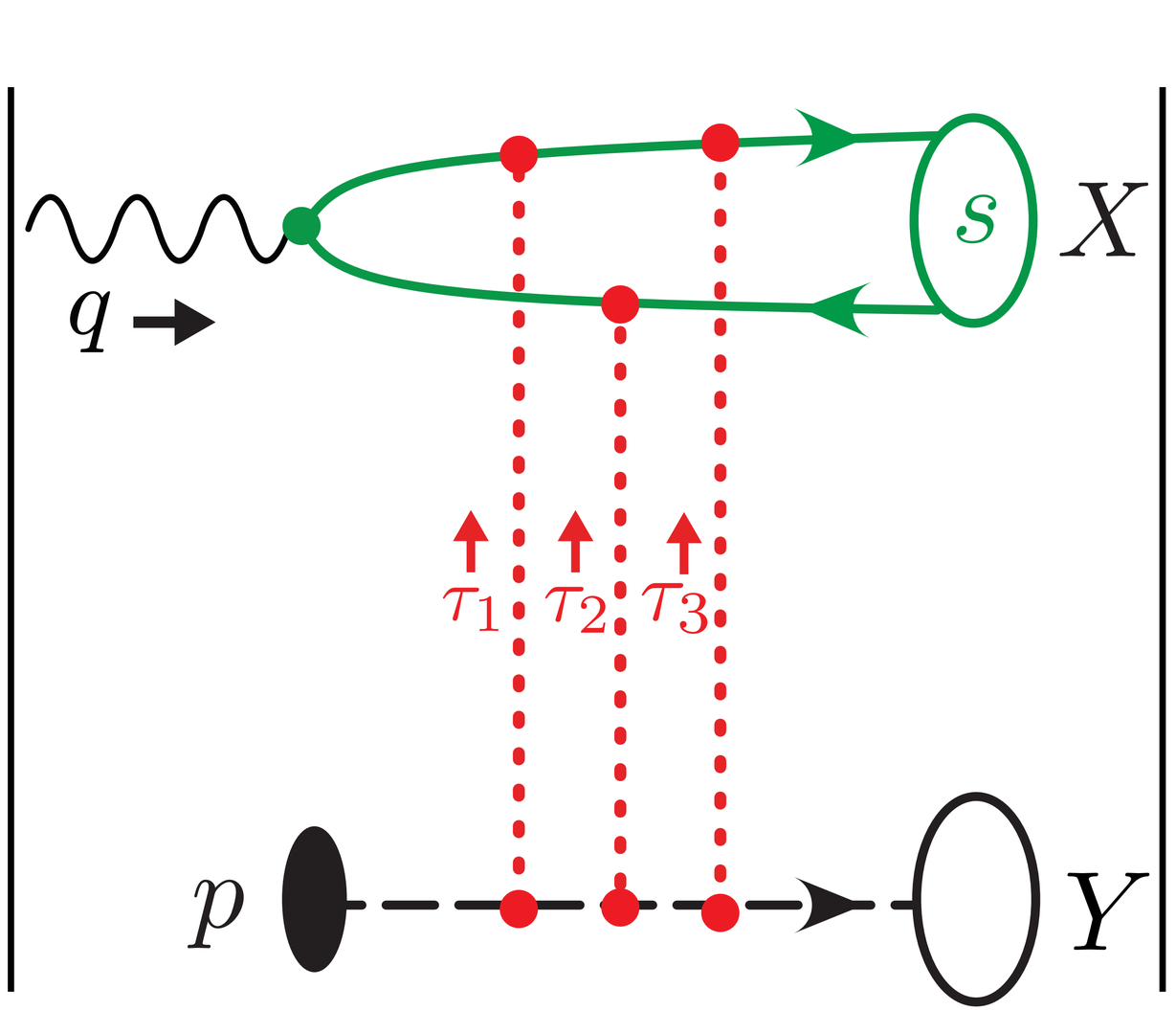


. . .

(Rothstein, Stewart, 1601.04695v3)

Glauber gluons

Rough factorization with only Glauber gluons



$=$

Soft

$$\left|\langle 0|J^\mu \mathcal{O}_s^{B_1}\mathcal{O}_s^{B_2}\mathcal{O}_s^{B_3}|X\rangle\right|^2$$

$\otimes_\perp$

$$\left(\tau_{1\perp}^{-2}\tau_{2\perp}^{-2}\tau_{3\perp}^{-2}\right)^2$$

$\otimes_\perp$

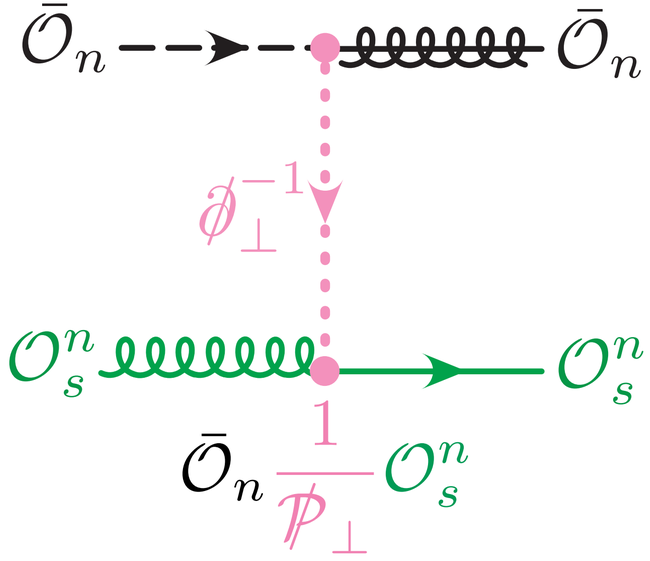
Beam

$$\left|\langle p|\mathcal{O}_n^{A_1}\mathcal{O}_n^{A_2}\mathcal{O}_n^{A_3}|Y\rangle\right|^2$$

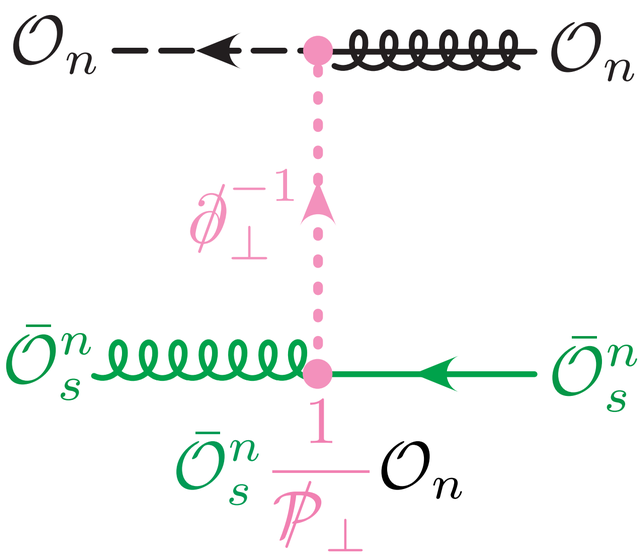
Glauber quarks

Sub-leading power ‘connections’ between sectors

$\mathcal{L}^{(1/2)}_{\text{Glauber}} \supset$



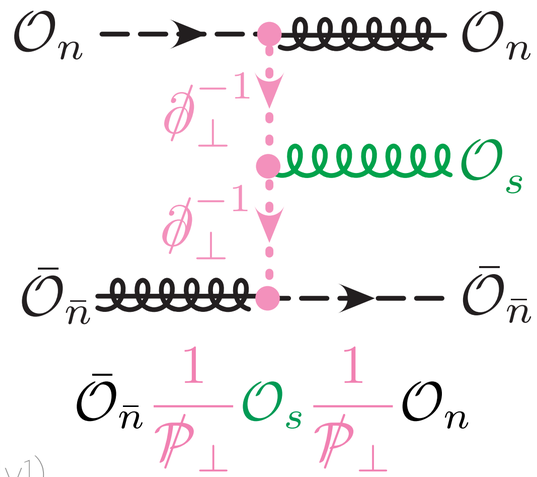
$+$



$+ \quad \bar{n}$ terms

$\mathcal{O}(\lambda^{1/2})$

$\mathcal{L}^{(1)}_{\text{Glauber}} \supset$

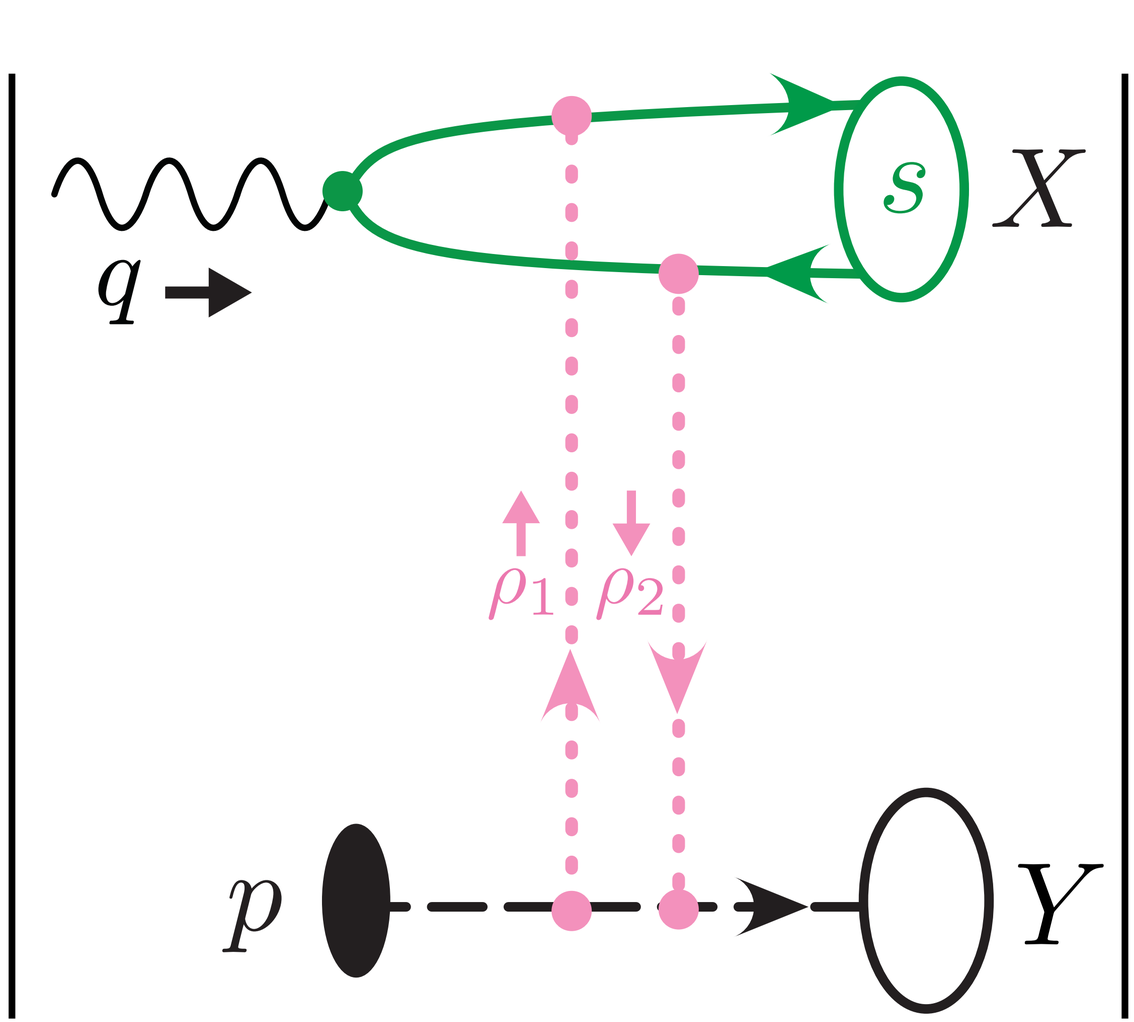


$\mathcal{O}(\lambda^1)$

(Moult, Solon,, Stewart, Vita,, 1709.09174v1)

Glauber quarks

Rough factorization with only Glauber quarks



$$= \Big((\not{p}_{1\perp}^{-1})_{\alpha\beta}(\not{p}_{2\perp}^{-1})_{\alpha'\beta'}\Big)^2$$

Soft

$$|\langle 0|J^\mu \mathcal{O}_s^\beta \bar{\mathcal{O}}_s^{\beta'}|X\rangle|^2$$

$\otimes_\perp$

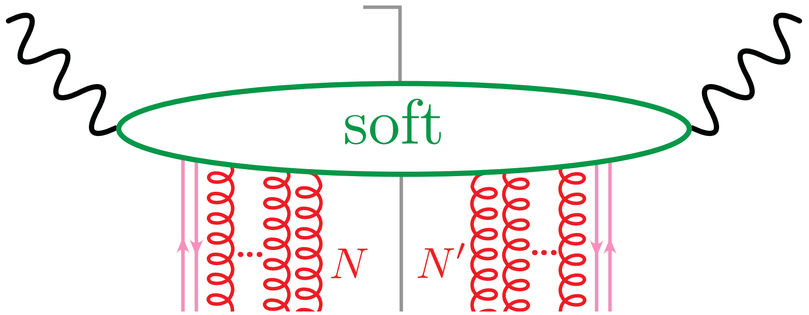
$\otimes_\perp$

Beam

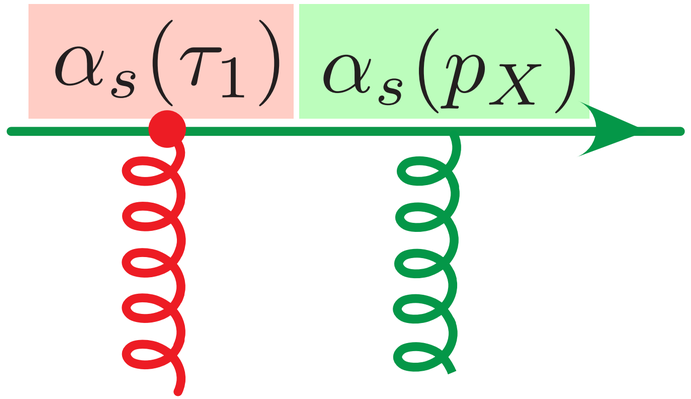
$$|\langle p|\bar{\mathcal{O}}_n^\alpha \mathcal{O}_n^{\alpha'}|Y\rangle|^2$$

Calculating LO Soft

$$-\sum_i \tau_i^2 = -t \gg \Lambda_{\rm QCD}$$



S can be perturbatively calculated

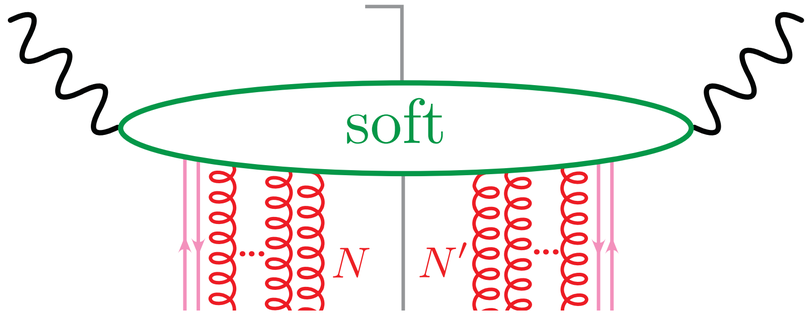


Types of strong couplings

soft:	"perturbative":	$\alpha_s(p_X) \ll 1$
Glauber:	"perturbative":	$\alpha_s(\tau_1 \gg \Lambda_{\rm QCD}) \ll 1$
	"non-perturbative":	$\alpha_s(\tau_1 \sim \Lambda_{\rm QCD}) \sim 1$

$\Rightarrow$ *LO: no soft couplings*

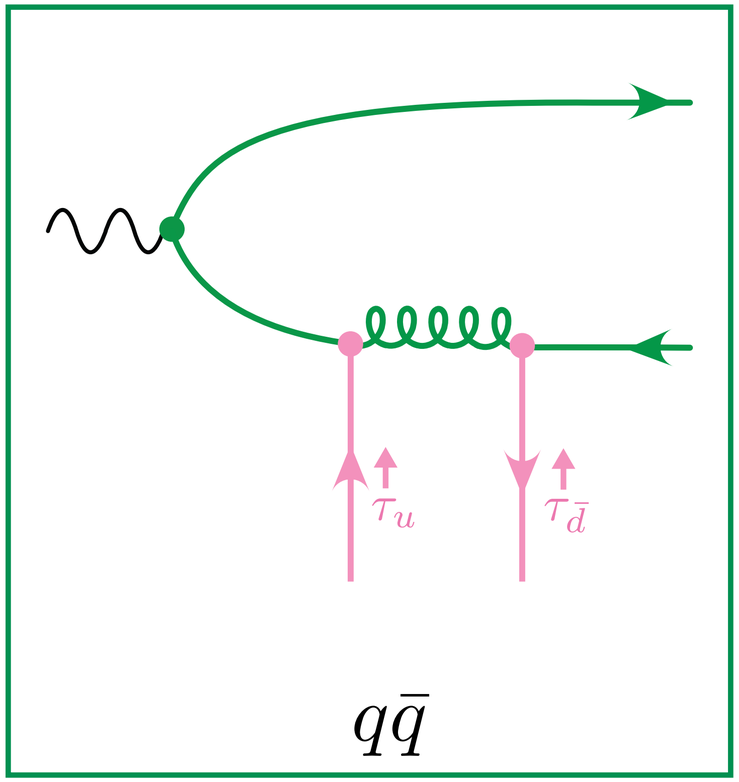
Calculating LO Soft

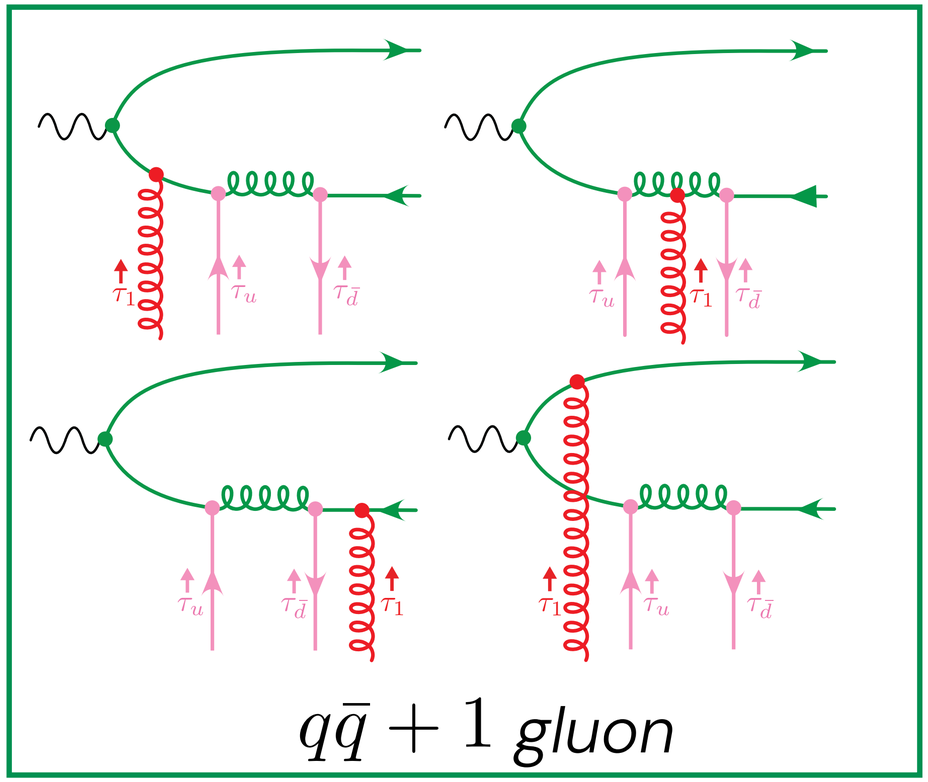


$$-\sum_i \tau_i^2 = -t \gg \Lambda_{\mathrm{QCD}}$$

S can be **perturbatively** calculated:

Additional Glauber exchanges are **not necessarily** suppressed, because their individual momenta can be **nonperturbative**.

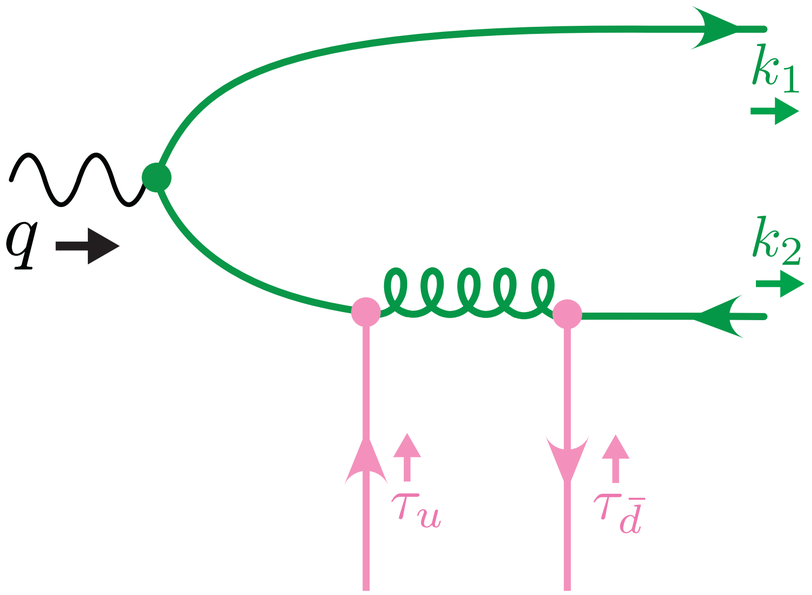




. . .

➡ ∞ number of Glauber gluons has to be included.

Calculating LO Soft



Soft function:

$$S_{q\bar{q}} \propto \frac{C_F^2}{(2k_2^+)^2}\left[\gamma_{\perp\alpha}(-\not{k}_2)\gamma_{\perp\alpha'}\gamma^0\right]_{uu'} \times\left[\gamma^0\gamma_\perp^{\alpha'}\frac{1}{\not{k}_2-\not{t}'}\gamma^\nu\not{k}_1\gamma^\mu\frac{1}{\not{k}_2-\not{t}}\gamma_\perp^{\alpha}\right]_{v'v}$$

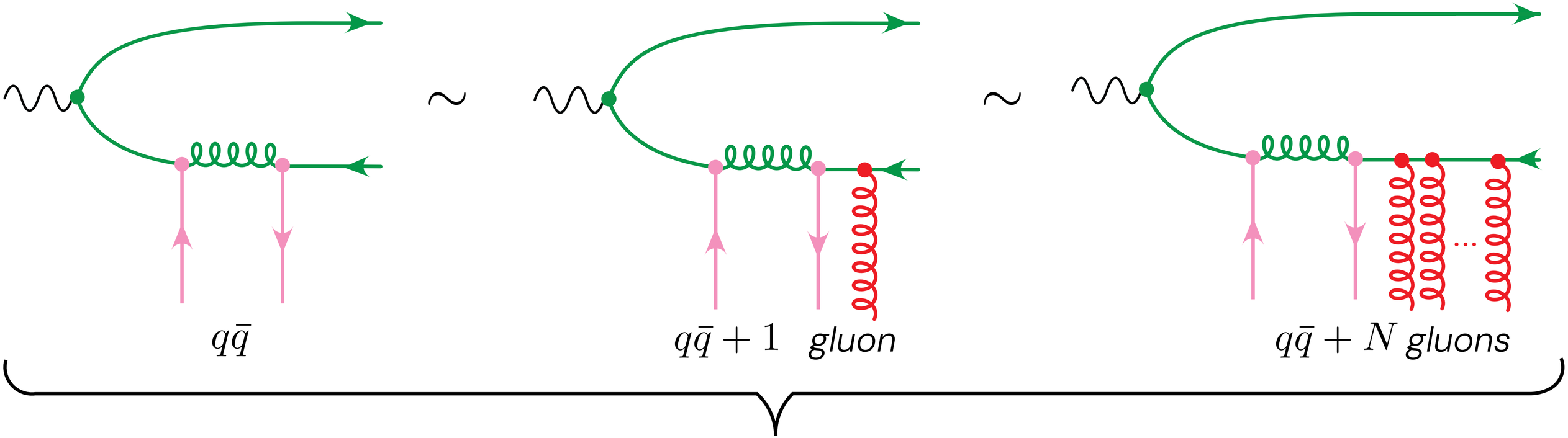
Final neutron is colorless, the exchange in done via a **color singlet**: $3\otimes\bar{3}\to 1$

Glauber fermions carry Dirac indices ⟶ **open spinor indices** $(uv;u'v')$

Calculating LO Soft

In the limit $\tau_1^2 \sim \tau_2^2 \sim \ldots \tau_N^2 \sim \Lambda_{\rm QCD}^2$ $(\tau_u + \tau_{\bar{d}} + \tau_1 + \cdots + \tau_N)^2 \gg \Lambda_{\rm QCD}^2$

individual gluon Glaubers (non-perturbative) total momentum transfer (perturbative)



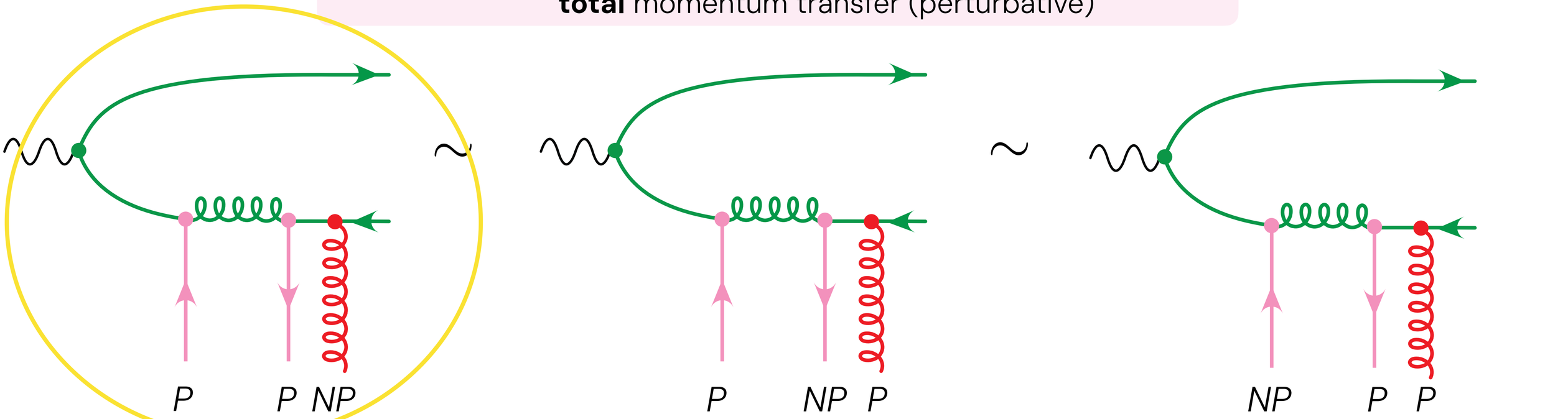
Same soft-function structure, the only difference is **color factor**

$$S_{q\bar{q}\text{-pert},g\text{-nonpert}} = S_{q\bar{q}} \times \text{Color factors}$$

Calculating LO Soft

In the limit $(\tau_u + \tau_{\bar{d}} + \tau_1 + \cdots + \tau_N)^2 \gg \Lambda^2_{\rm QCD}$

total momentum transfer (perturbative)



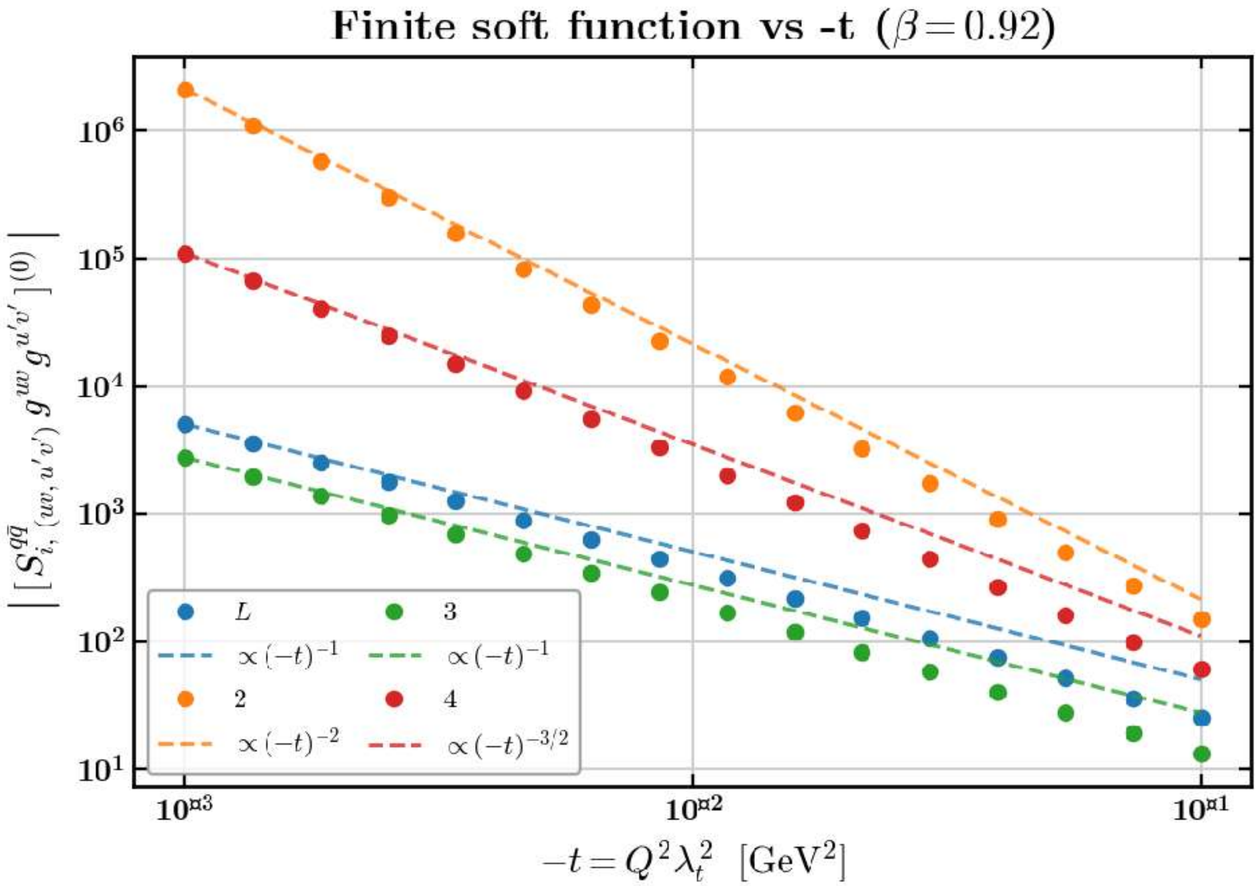
Only this one simplifies

Same order, different structures!

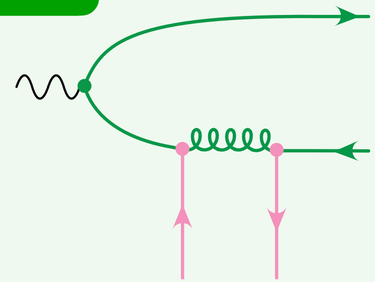
Full Soft function $\neq S_{q\bar{q}} \times$ Color factors

Not a pion!

Soft function



Toy example



- **LO soft function**, only Glauber quarks
- **Simplified case:** spin-blind interaction (free spinor indices are fully contracted)
- All polarized, asymmetric structure functions = 0; only L, 2, 3, 4 remain

Scalings: $S_L \sim \lambda_t^{-2}, \quad S_2 \sim \lambda_t^{-4}, \quad S_3 \sim \lambda_t^{-2}, \quad S_4 \sim \lambda_t^{-3}$

$$\lambda_t = \frac{\sqrt{-t}}{Q}$$

Problems: a very simple case; in reality will have a model for the jet function or at least a Dirac-basis expansion of it. With a good model the computation is doable.

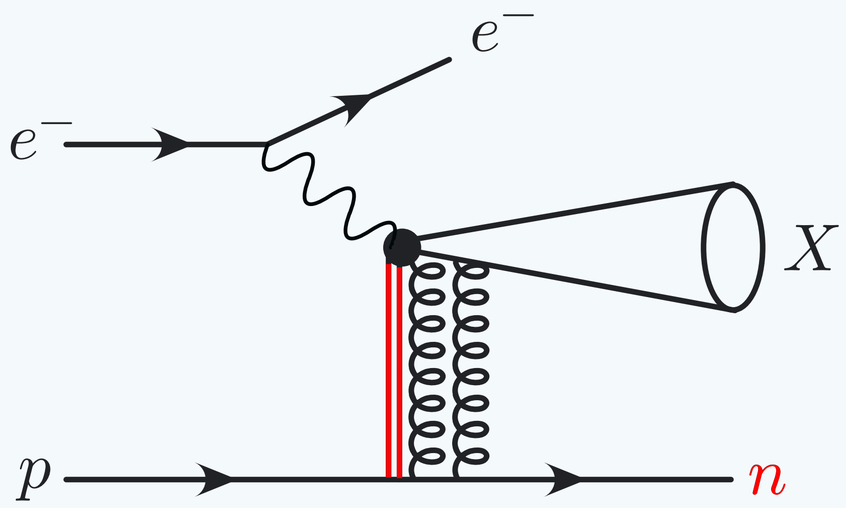
Scalings of structure functions

- We can estimate structure function sizes (their scaling with λ):

Forward neutron

$$F_i^D(ep \to eXn) \sim \lambda^{-4}$$

$$\mathcal{L}_{\text{Glauber } q} \sim \lambda^{1/2}$$

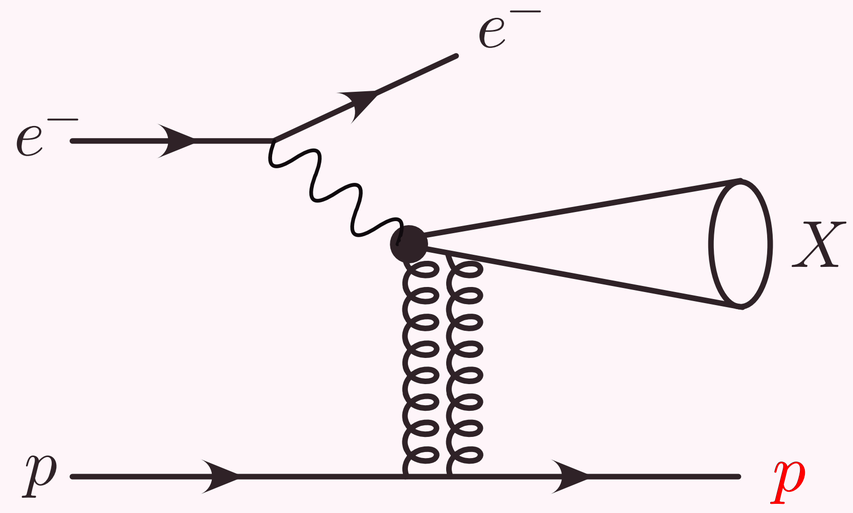


more suppressed!

Forward proton

$$F_i^D(ep \to eXp) \sim \lambda^{-6}$$

$$\mathcal{L}_{\text{Glauber } g} \sim \lambda^{0}$$



less suppressed!

Spin asymmetries

Asym.	Full eighteen-function sources	Source of target spin dependence in $W_D^{\mu\nu}$	Tensor symmetry
A_{LU}	F_{4A}^D	none	antisymmetric
A_{UL}	F_5^D, F_7^D through $\bar{U}_L^\mu$	$\bar{U}^\mu = \bar{U}_L^\mu + \bar{U}_T^\mu$	symmetric
A_{UT}	F_{LP}^D, F_{2P}^D, F_{3P}^D, F_{4P}^D, and F_5^D, F_6^D, F_7^D, F_8^D through transverse-spin pieces	$\widetilde{S}_T' = S_T \sin(\phi - \phi_S)$ for F_{LP}^D, F_{2P}^D, F_{3P}^D, F_{4P}^D; $\bar{U}_T^\mu$ and $\bar{X}_T^\mu$ for F_5^D, F_6^D, F_7^D, F_8^D	symmetric
A_{LL}	F_{5A}^D, F_{7A}^D through $\bar{U}_L^\mu$	$\bar{U}^\mu = \bar{U}_L^\mu + \bar{U}_T^\mu$	antisymmetric
A_{LT}	F_{4AP}^D, and F_{5A}^D, F_{6A}^D, F_{7A}^D, F_{8A}^D through transverse-spin pieces	$\widetilde{S}_T' = S_T \sin(\phi - \phi_S)$ for F_{4AP}^D; $\bar{U}_T^\mu$ and $\bar{X}_T^\mu$ for F_{5A}^D, F_{6A}^D, F_{7A}^D, F_{8A}^D	antisymmetric

Table 1: Spin-asymmetry classification in the full eighteen-function covariant diffractive basis. The table lists the structure functions that can contribute to each spin asymmetry, together with the corresponding source of spin dependence in $W_D^{\mu\nu}$ and the tensor symmetry selected by the lepton-side contraction.

Extensions

Additional tagged particles:

$$S_{\text{inc}} \to S_{\text{semi-inc}} \sim \int \langle 0|...|X'_s\pi\rangle\langle X'_s\pi|...|0\rangle$$

Neutron vs. Lambda 0:

Jet:

$$J_n = \langle p|\{\!\{\mathcal{O}_n^{\text{up}}\mathcal{O}_n^{\overline{\text{down}}}\prod\mathcal{O}_n^{\text{gluons}}\}\!\}|n\rangle \quad \to \quad J_{\Lambda^0} = \langle p|\{\!\{\mathcal{O}_n^{\text{up}}\mathcal{O}_n^{\overline{\text{strange}}}\prod\mathcal{O}_n^{\text{gluons}}\}\!\}|\Lambda^0\rangle$$

Convolution: $\not{t}_{\bar{d}\perp} \to \not{t}_{\bar{s}\perp} - m_s$

In general for other forward hadrons:

$$J_n = \langle p|\{\!\{\mathcal{O}_n^{\text{up}}\mathcal{O}_n^{\overline{\text{down}}}\prod\mathcal{O}_n^{\text{gluons}}\}\!\}|n\rangle \quad \to \quad J_h = \langle h|\{\!\{\underbrace{\mathcal{O}_n^{q_1}\mathcal{O}_n^{q_2}...\mathcal{O}_n^{q_N}}\prod\mathcal{O}_n^{\text{gluons}}\}\!\}|h'\rangle$$

min. set of quarks to transform the initial hadron to the final one

Pion model

$$|p\rangle = \sqrt{Z}|p_0\rangle + a\,|n\pi^+\rangle + b\,|p\pi^0\rangle + c\,|\Delta\pi\rangle + \cdots$$

$$\underbrace{p \to n + \pi}_{\text{long-distance/chiral physics}} \quad \otimes \quad \underbrace{\gamma^*\pi \to X}_{\text{short-distance DIS}}$$

$$F_2^{\text{Sullivan}}(x, Q^2, x_L, t) = f_{\pi/p}(x_L, t) F_2^{\pi}\left(\frac{x}{1-x_L}, Q^2\right)$$

$$f_{\pi/p}(x_L, t) = \frac{2g_{\pi pp}^2}{(4\pi)^2} \frac{-t}{(t-m_\pi^2)^2} (1-x_L)^{1-2\alpha_\pi(t)} F(x_L, t)^2$$

$$|\mathcal{M}|^2 \sim \frac{|\mathcal{M}_{\gamma^*\pi\to X}|^2}{(t-m_\pi^2)^2}$$

near pion pole: reaction becomes dominated by a nearly on-shell pion

But we cannot kinematically reach the pole. People use extrapolation.

Pion model

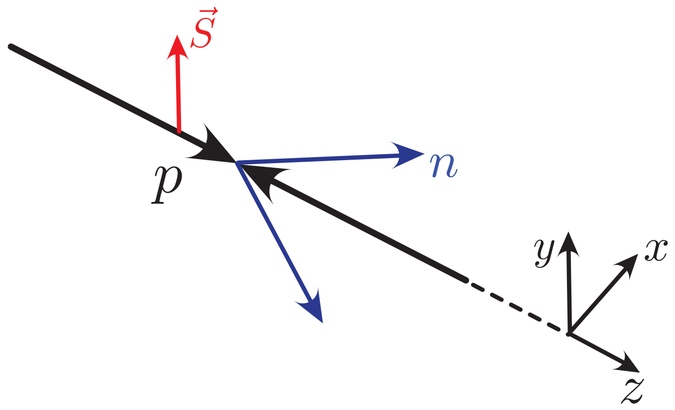
The biggest model dependence comes from:

- F **vertex form factor** – describes how the $p \to n\pi$ vertex weakens for highly virtual pions.
- **Cutoff scale** Λ – controls how quickly the form factor suppresses large momentum or virtuality.
- **Off-shell effects** – the exchanged pion is virtual, so its properties may differ from those of a free physical pion.
- $\Delta\pi$ **components** – the proton can also fluctuate into a $\Delta + \pi$ state, rather than only an $N + \pi$ state.
- **Reggeized vs. simple pion exchange** – different theoretical descriptions of pion exchange, especially at higher energies.
- **Absorption / final-state interactions** – produced particles can interact with the spectator neutron, modifying the measured cross section.
- ρ, a_2, ... **exchanges** – other mesons can also be exchanged and can contaminate or mimic the pion-exchange signal.

Pion model

Pion does not **create enough spin asymmetry.**

People try to model difference with absorptive corrections or extra resonances.



$$A_N \propto \frac{N_L - N_R}{N_L + N_R}$$

$$A^B_{p\to n}(\vec{q}, z) = \bar{\xi}_n \left[\sigma_3\, q_L + \frac{1}{\sqrt{z}}\, \vec{\sigma}\cdot\vec{q}_T\right] \xi_p\, \phi^B(q_T, z)$$

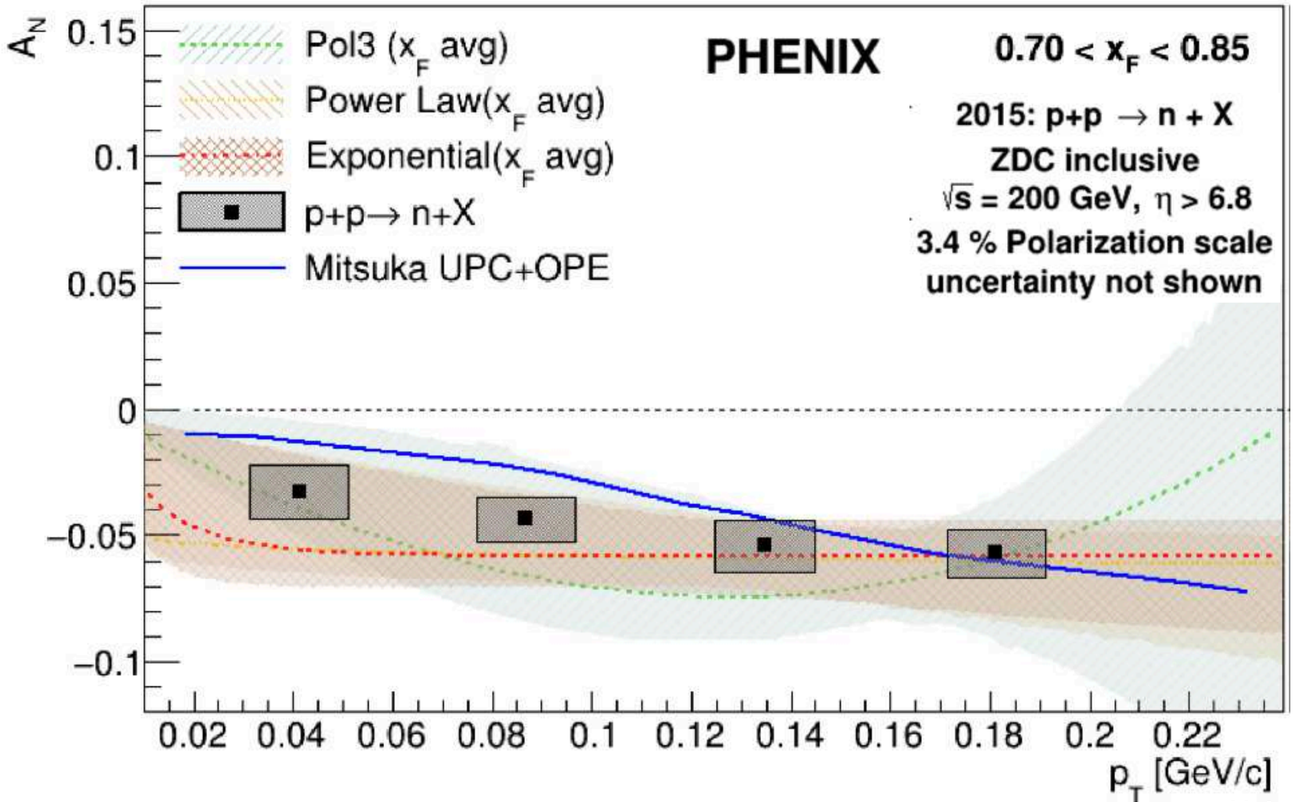
spin non-flip spin-flip

$$A_N \sim \sin(\delta_{\text{flip}} - \delta_{\text{non-flip}})$$

phase difference of the flip/non-flip

$$\delta_{\text{flip}} = \delta_{\text{non-flip}} \Rightarrow A_N = 0$$

Prediction: no single-transverse spin asymmetry



(Kopeliovich, Potashnikova, Schmidt, Soffer 0805.4534, 1109.2500)

(PHENIX 2110.07504)